

**EFFECT OF APPLICATION OF COOPERATIVE LEARNING MODEL
OF *THINK TALK WRITE* TYPE ON UNDERSTANDING THE
MATHEMATICAL CONCEPT OF STUDENTS IN GRADE XI SMA
ADABIAH 2 PADANG**

THESIS

To meet some of the requirements for a Bachelor of Education degree



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ABSTRACT

Vanya Aridanthy : Effect Of Application Of Cooperative Learning Model Of *Think Talk Write* Type On Understanding The Mathematical Concept Of Students In Grade XI Sma Adabiah 2 Padang

One of the objectives of mathematics learning is understanding mathematical concepts. But the reality is that in SMA Adabiah 2 Padang this ability is still not optimal. Learning strategies have not been able to improve the ability to understand the concept of learners. If this ability is low, it will affect other mathematical abilities. The purpose of the study was to analyze the improved understanding of the concept of learners learning with the *Think Talk Write* learning model and describe whether understanding the concept of learners learning with the *Think Talk Write* learning model is better than understanding the concept of learners learning with direct learning models in grade XI MIA SMA Adabiah 2 Padang Year Lesson 2019/2020.

This type of research is *quasy experiment* and descriptive with the design of *Randomized Control Group Only Design* research. The population is class XI MIA SMA Adabiah 2 Padang with research samples namely class XI MIA 1 as experimental class and XI MIA 2 as control class. Research instruments in the form of quizzes and concept comprehension tests.

Data on the development of mathematical concept understanding ability is described based on the results of quizzes, LKPD, and exercises, while the test results are analyzed with t-tests and obtained $P\text{-value} = 0.003$. Based on the results of the quiz, it was obtained that the development of concept understanding increased with the *Think Talk Write* learning model, and based on the results of the test analysis obtained that the understanding of the concept of learners who learned with the learning model of *Think Talk Write* is better than understanding the concept of learners who learn with the direct learning model in grade XI MIA SMA Adabiah 2 Padang.

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CHAPTER I INTRODUCTION

A. Background Problems

Entering the 21st century, the role of information and communication technology is very rapidly developing. The rapid development of information and communication technology is based on the development of mathematics. In order not to compete with other countries, mastery of mathematics early on becomes the solution, because mathematics is the basic science that underlies the development of information and communication technology. Mathematics plays an important role in advancing the thinking of learners. Therefore, through mathematics learning, students can practice thinking patterns in a critical, logical, creative, and systematic way.

The importance of mathematics is demonstrated by the use of mathematics as a compulsory subject from elementary school to college. The higher the level of education, the subject matter learned demands a higher level of thinking so that students are asked to be able to develop their mathematical skills.

Mathematics is one of the fields of study that has an important role in the world of Indonesian education. Soedjadi (2000:101) states that mathematics is a vehicle for teachers to bring learners to their goals. In the regulation of the Minister of Education and Culture No. 59 of 2014, there are eight objectives of mathematics learning that must be achieved, one of the objectives is so that students can understand the concept of mathematics. Good understanding of mathematical concepts, useful for learners to be able to achieve other mathematical learning objectives and will affect the learning outcomes of learners later. If the learner does not understand the concept of one then it is difficult for the learner to

understand the next concept so that it can result in the learning results of the learner has not been satisfactory. This also results in a decrease in the motivation of learners in learning mathematics, a lack of curiosity of learners towards the material taught and learners always assume that mathematics is a difficult lesson.

As a party that is directly involved in learning in the classroom, teachers have an important role to play in helping learners achieve math learning goals. Therefore, math teachers should strive to prepare lessons that are in accordance with the characteristics of the learners, the materials taught and the condition of the school so that the objectives of mathematics learning can be achieved.

Based on the observations that have been done on July 15 to July 27, 2019 and during the *Teacher training* process at SMA Adabiah 2 Padang, which has been using the 2013 curriculum, there are several phenomena related to mathematics learning, namely that students only accept what is said by educators so that communication that occurs only one way, and also during the learning process, educators give a problem or problem but the learners only wait for answers and copy their friends. Whereas in SMA Adabiah 2 Padang has implemented the 2013 curriculum but the implementation process has not been carried out optimally. As a result, learners are unable to build on the concept of the material studied. All of the above is expected to have an impact on the low understanding of the mathematical concepts of learners to the learning materials.

Based on the results of daily tests of students in grade XI MIA SMA Adabiah 2 Padang on trigonometric equations material, it appears that some of the questions given by teachers are about understanding mathematical concepts, but students still have difficulty answering them. Seen from the

learning results of learners who are under the *Minimum Criteria Of Mastery Learning* is 75. This can be seen in Table 1.

Table 1. Percentage Results of Daily Mathematics Test Concentration Grade XI MIA SMA Adabiah 2 Padang Year Lesson 2019/2020

No	Concept Comprehension Indicators	Concept Understanding Percentage Learners (%)
1	Reassess the concept learned	35,29
2	Classify objects based on filled or not requirements that make up the concept	30,58
3	Identify the properties of an operation or concept	32,94
4	Apply concepts logically	21,17

Source : Results Of Daily Mathematics Test sma Adabiah 2 Padang

In Table 1 above, it appears that the percentage of complete learning outcomes of learners has not been satisfactory, namely the percentage of students who complete less than 50%. This indicates that the understanding of the mathematical concepts of learners is still low. Based on the previous explanation, that if the learner does not understand a concept, it will affect the learner to achieve other mathematical goals and will affect the results of his/her learning. This can be seen from some of the students' daily replay questions. The daily replay question is a question of understanding mathematical concepts. From the answer sheet of the learners, there are still many students who have problems in finding solutions to the given questions. The following is explained by the student's answer sheet from the test question. The problems given are as follows:

Determine Equation Resolution $2 \cos(x - 20^\circ) - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$!

The learner's answer sheet can be seen in Figure 1 and Figure 2. In Figure 1 and Figure 2, it appears that learners still have difficulty in solving trigonometric

equations related to indicators applying concepts logically and identifying the properties of concepts.

Figure 1 shows handwritten mathematical work for two trigonometric equations. The first equation is $\cos(x-20^\circ) = 0$, which is solved by $x-20^\circ = 90^\circ$ and $x-20^\circ = 270^\circ$, leading to $x = 110^\circ$ and $x = 290^\circ$. The second equation is $\cos(x-20^\circ) = \frac{1}{2}$, which is solved by $x-20^\circ = 60^\circ$ and $x-20^\circ = 300^\circ$, leading to $x = 80^\circ$ and $x = 320^\circ$. The final answer set is $\{110^\circ, 290^\circ, 80^\circ, 320^\circ\}$.

Figure 1. Learner A Answers results for Indicators 3 and 4

Figure 2 shows handwritten mathematical work for two trigonometric equations. The first equation is $\cos(x-20^\circ) = \frac{1}{2}$, which is solved by $x-20^\circ = 60^\circ$ and $x-20^\circ = 300^\circ$, leading to $x = 80^\circ$ and $x = 320^\circ$. The second equation is $\cos(x-20^\circ) = \frac{1}{4}$, which is solved by $x-20^\circ = 75^\circ$ and $x-20^\circ = 285^\circ$, leading to $x = 95^\circ$ and $x = 305^\circ$. The final answer set is $\{80^\circ, 320^\circ, 95^\circ, 305^\circ\}$.

Figure 2. Learners B Answer Results for Indicators 3 and 4

Based on the picture above, it is known that learners have difficulty in applying concepts logically and identifying the properties of concepts that should be used and linking them to mathematical concepts and outside mathematics, namely the concept of trigonometric comparison. The concept on cos that should be used is

$$x = \alpha^\circ + k \cdot 360^\circ \text{ and } x = -\alpha + k \cdot 360^\circ.$$

But learners use the concept in sin. So that the learner is wrong in re-stating the concept studied.

Alternative expected answers are as follows:

$$2 \cos(x - 20^\circ) - 1 = 0 \quad \text{for } 0^\circ < x < 360^\circ$$

Solving :

$$2 \cos(x - 20^\circ) - 1 = 0$$

$$\cos(x - 20^\circ) = \frac{1}{2}$$

$$\cos(x - 20^\circ) = \cos 60^\circ$$

➤ Formula 1

$$x = a + k \cdot 360^\circ$$

$$x - 20^\circ = 60^\circ + k \cdot 360^\circ$$

$$x = 60^\circ + 20^\circ + k \cdot 360^\circ$$

$$x = 80^\circ + k \cdot 360^\circ$$

$$k = 0 \rightarrow x = 80^\circ$$

$$k = 1 \rightarrow x = 440^\circ$$

➤ Formula 2

$$x = -a + k \cdot 360^\circ$$

$$x - 20^\circ = -60^\circ + k \cdot 360^\circ$$

$$x = -60^\circ + 20^\circ + k \cdot 360^\circ$$

$$x = -40^\circ + k \cdot 360^\circ$$

$$k = 0 \rightarrow x = -40^\circ$$

$$k = 1 \rightarrow x = 320^\circ$$

$$k = 2 \rightarrow x = 680^\circ$$

The solution set is

$$\{80^\circ, 320^\circ\}$$

Understanding the low math concepts of learners is also seen from the following daily test questions

Define an equation solution $\sqrt{3}\tan x = 1$ for $0 < x < 3\pi$!

The learner's answer sheet can be seen in Figure 3 and Figure 4. In Figure 3 and Figure 4, it appears that students still have difficulty in solving trigonometric equations related to indicators classifying objects according to their properties according to their concept.

Figure 3. Learner Answer Results C for Indicators 1 and 2

Figure 4. Learner D Answer Results for Indicators 1 and 2

Pada In Figure 3, students found difficulty in identifying the properties of operations or concepts and indicators classifying objects according to their properties according to their concept, as seen in the picture the learners used the cosine equation formula to solve the problem, but the concept that should be used is the concept of tan equation. Likewise with Figure 4, students also use the cosine equation formula to solve the problem.

Alternative expected answers are as follows:

$$\sqrt{3} \tan x = 1 \text{ for } 0 < x < 3\pi$$

Solving :

$$\sqrt{3} \tan x = 1$$

$$\tan x = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan x = \frac{1}{3} \sqrt{3}$$

$$\tan x = \tan 30^\circ$$

$$\left(\frac{1}{6} \pi \right)$$

Formula $x = a + k \cdot \pi$

$$x = \frac{1}{6} \pi + k \cdot \pi$$

$$k = 0 \rightarrow x = \frac{1}{6} \pi$$

$$k = 1 \rightarrow x = \frac{7}{6} \pi$$

$$k = 2 \rightarrow x = \frac{13}{6} \pi$$

The solution set is $= \left\{ \frac{1}{6} \pi, \frac{7}{6} \pi, \frac{13}{6} \pi \right\}$

Based on some of the above questions and the results of daily replays of students showed that the understanding of the mathematical concepts of students of Adabiah 2 Padang High School still needs to be improved. Based on the observations, educators have also tried to design a Learning Implementation Plan (RPP) that refers to the 2013 Curriculum, namely by applying direct learning and scientific approaches. However, the teaching and learning process is not yet fully in accordance with the RPP that has been designed. After the educator prepares the learner to learn as well as explain the importance of the lesson, then the educator presents the information gradually and the learner enthusiastically observes and enthuses. But for reasoning activities, gathering information and communicating has not been carried out optimally.

The low understanding of students' mathematical concepts makes it difficult for students to solve math concepts. The difficulties experienced by learners are caused because the learning process is still centered on teachers and learners are still less active during the learning process. Although the teacher had given the opportunity, no students asked. As a result, students have difficulty understanding the concept. In addition, during the learning process, researchers noticed that some students did not pay attention to learning. Learners are busy with their own activities that are not related to the learning process such as, telling stories that are not related to the material, disturbing their next-door friends and drawing unnecessary things.

In order for the understanding of the mathematical concepts of learners to develop optimally, learners must be active in the learning process. Active learners will make it easier to understand the materials studied. One of the efforts that teachers can make is to design lessons that are centered on learners, so that the students themselves are actively involved in building their knowledge. In accordance with Regulation of the Minister of Education and Culture No. 22 of 2016 "Curriculum 2013 requires the learning process using a scientific approach. In the learning process using a scientific approach, learners observe, ask, try, reason, and communicate".

Scientific approach can be carried out optimally by using various learning models. One of the learning models that can be used with a scientific approach is a cooperative learning model. Pratt (Jufri, 2013:112) states that cooperative learning is essential to guide and facilitate learners in the learning process. This opinion is reinforced by the Ministry of Education (2016:355) that with cooperative learning, students can help each other in the learning process, so that later learners can optimize their understanding with the help of their friends. One of the cooperative learning models that can be used with a scientific approach is the *Think Talk Write* type cooperative learning model.

Think Talk Write type cooperative learning model is estimated to match the characteristics of students at SMA Adabiah 2 Padang who are still less active in learning. This is because in the *Think Talk Write* model it is given time to think individually first, draw up ideas or ideas and then write down those ideas. Students then speak or share ideas or ideas in group discussions and continue to write ideas or ideas that they get in the form of reports or conclusions. In group discussions, students interact and collaborate with groupmates to discuss materials or issues provided. Understanding is built by the learners through interaction in the discussion, then the learners channel it into the form of writing. In the final activity of learning is to make reflections and conclusions on the material given.

Table 2. The Linkage of *Think Talk Write* Syntax with Concept Understanding Indicators

Syntax	Concept Comprehension Indicators
Syntax 1 <i>Think</i>	Classify objects based on the requirements that make up the concept, identify the properties of operations or concepts, and relate various concepts in mathematical concepts as well as outside mathematics
Syntax 2 <i>Talk</i>	Reasate a concept that has been studied

Syntax 3 <i>Write</i>	logically apply concepts, and present concepts of various forms of mathematical representation (tables, graphs, diagrams, sketches, mathematical models or any other way)
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Through this learning model, it is expected that students can actively participate in finding concepts and have more opportunities in building their knowledge both individually and in groups. Based on the background of the problem that has been described, the author conducted a study with the title: **"The Effect of Application of Cooperative Learning Model Type *Think Talk Write* On Understanding the Mathematical Concept of Students Grade XI SMA Adabiah 2 Padang"**

B. Identify Problems

Based on the background above can be identified some problems in grade XI MIA SMA Adabiah 2 Padang in mathematics learning as follows:

1. Learners are still less active in learning
2. Understanding the mathematical concepts of learners is still low
3. The learning process is still teacher-centered
4. The learning model used by teachers has not varied

C. Problem Limitation

Based on the background and identification of the problems described above, the limitations of the problem in this study are the understanding of the math concepts of learners is still low and the learning model used by teachers has not varied. This issue will be addressed by implementing a Think Talk Write type cooperative learning model.

D. Problem Formulation

Referring to the limitations of the problem presented, the problem formulation in this study is as follows:

1. Is there a difference in understanding the mathematical concepts of learners who learn by using the *Think Talk Write* type cooperative learning model and who learn using direct learning in class XI MIA SMA Adabiah 2 Padang?.
2. How is the development of understanding the mathematical concept of Learners who use *Think Talk Write* learning in class XI MIA SMA Adabiah 2 Padang?."

E. Research Objectives

Based on the problems that have been raised, this research aims to:

1. Analyze and describe the mathematical concept understanding of learners whose learning uses Think Talk Write and compare it with students who study directly in class XI MIA SMA Adabiah 2 Padang.
2. Knowing and describing how the development of understanding the concept of learners using Think Talk Write learning in class XI MIA SMA Adabiah 2 Padang.

F. Research Benefits

1. The expected benefits of this research are: For researchers, as a provision of knowledge teaching mathematics in schools later, especially in the use of think talk write models and understanding mathematical concepts.

2. For learners, in addition to learning experience to be able to improve understanding of mathematical concepts and to be more active and active in learning so as to improve learning outcomes.
3. For math teachers, in order to give an overview of the application of think talk write approach in helping to choose the appropriate learning methods to improve the understanding of mathematical concepts of learners.
4. For the headmaster, it can be used as a consideration and evaluation to achieve a better quality of education.
5. For other researchers, as a source of information to conduct more in-depth research on the problems of this research

CHAPTER II THEORETICAL STUDIES

A. Theoretical Studies

1. Cooperative Learning Model

a. Pen Understanding cooperative learning models

Cooperative learning according to Rusman (2012: 202) is a form of learning by way of learners learning and working in small groups collaboratively whose members consist of four to six people with *heterogeneous* group structure. Furthermore, according to Smith (Conring: 2009) states that cooperative learning is a learning model in which learners work in groups with conditions that meet the criteria, namely positive interdependence, individual accountability, face-to-face interaction, proper use of collaborative skills, and regular self-assessment of functioning teams.

Based on this opinion, it can be concluded that the cooperative learning model is a form of learning in small groups that is carried out through process sharing between learners, so that there is a common understanding and a broad interaction between the learners themselves. Cooperative learning is in constructivist theory. This learning arises from the concept of constructivism that students will more easily understand difficult mathematical concepts if they discuss with their friends. Learners routinely work in groups to help each other solve problems.

According to Isjoni (2009: 17) in conducting the cooperative learning process teachers no longer dominate as usual, so that learners are required to share information with other learners and learn to teach each other. The advantages of this model are in helping learners understand difficult concepts and to foster critical thinking skills.

In cooperative learning, students are formed in groups of 4 to 6 students who are equal but heterogeneous, to work together to master the material and to complete the tasks that the teacher has given. Group members have a responsibility and depend on each other to achieve common goals.

b. Objectives of Cooperative Learning Model

The purpose of the establishment of cooperative groups is to provide opportunities for learners to actively engage in thought processes and learning activities, while the main objective of cooperative learning according to Johnson in Trianto (2012: 57) is to maximize the learning of learners to improve academic achievement and understanding both individually and in groups. Learners work together as a team, thus improving relationships between students from different ethnic, cultural and ability backgrounds, and can also develop group process skills and problem solving.

c. Syntax of Cooperative Learning Activities

According to Suprijono (2013: 65) the cooperative learning model synth consists of six stages

Table 3. Cooperative Learning Activity Syntax

Stage	Teacher's Behavior
Stage 1 Delivering goals and preparing learners	The teacher classifies the purpose of cooperative learning. This is important to do because learners must clearly understand the procedures and rules in learning
Stage 2 Present information	The teacher conveys information, because this information is academic content
Stage 3 Organizing learners to in study groups	The teacher explained that students should work together in groups. Completion of a group task must be the task of the group itself
Stage 4 Help with group work and learning	The teacher accompanies the study groups, reminding them of the tasks the learners are doing and the time allocated
Stage 5 Evaluation	Teachers conduct evaluations using evaluation strategies that consistent with learning objectives
Stage 6 Give recognition or appreciation	Teachers prepare rewards to be given to learners

Based on the above opinions, cooperative learning model is a group learning activity that involves the participation of learners in one small group that interact with each other. Each group consists of different abilities, but all group members work together and help each other to solve problems by applying knowledge and skills.

2. *Think Talk Write Learning Model*

The Think Talk Write learning model was first introduced by Huinker and Laughlin in 1996 (Huda 2014: 218). This model is based on the understanding that learning is social behavior. In this learning model, learners are encouraged to think, talk, and then write about a topic. This method is a method that can train the thinking and speaking skills of learners. This is in line with Suminar's opinion (2015: 300) which states that the TTW model can make students more active in class and learners feel relaxed if they cooperate or learn cooperatively, learners can think about what they know and they share with the group and then write on the paper after they discuss with their group. In this TTW learning model, students will learn in groups of 3-5 people. In this group, students are asked to explain and share ideas with friends so that students can re-write down the solution to the problem in their own language.

This TTW learning model has 3 stages of learning, namely:

a. Think

Thinking is the development of ideas and concepts within a person that involve the work of the brain. In Kamus Besar Bahasa Indonesia (KBBI) thinks it means using reason to consider and decide things, as well as to weigh in memory.

This think or thinking stage can be seen through the process of reading a mathematical text, thinking about possible answers, and then making notes of what has been read. When creating or writing these notes learners are required to be able to distinguish and unite the ideas presented in the reading text to be translated into their own language.

At this stage the learner will read a number of problems given on the *Student Worksheet*, then after reading the learner will write down the known and unknown things about the problem (making individual notes). Furthermore, students are asked to solve existing problems individually. The thought process at this stage will be seen when the learner reads the problem and then re-writes down what is known and unknown about a problem. In addition, the thought process will occur when learners attempt to solve problems in the *Student Worksheet* individually.

According to Marzuki in Elida (2012: 181-182) that human thinking includes five dimensions, namely:

1. Metacognition, is a person's awareness of his or her thought process when doing what is done.
2. Critical and creative thinking, are two very basic components. Critical thinking is the process of using the ability to think effectively that can help a person to make, evaluate, and make decisions about what is believed and done. While creative thinking is a spontaneous ability, occurs because of internal directives and its existence is unpredictable.
3. The thought process, has several main components namely concept formation, principle formation, understanding, problem solving, decision making, research and preparation.

4. Main thinking ability, has eight components that focus, ability to get information, ability to remember, ability to organize, ability to analyze, ability to produce, ability to integrate, and ability to evaluate.
5. High-level mathematical thinking, is essentially non-procedural which among others includes the following: the ability to find and explore patterns, the ability to use facts, the ability to make mathematical ideas, the ability to think and reason flexibly, and establish that a problem solving is logical.

b. Talk

Talk is defined as talking or discussing. At this stage the learners communicate using their own words and language. The talk stage allows learners to be skilled at speaking. At this stage the learner will practice doing mathematical communication with members of his group orally. The issue that will be discussed is an issue that learners have thought about before at the think stage.

The talking process is learned by learners through their lives as individuals who interact with the social environment. By discussing it can increase the activities of learners in the classroom. Communicating in discussions creates a learning environment that encourages learners to communicate between other learners and can improve the understanding of learners because when students discuss, learners construct various ideas to put forward.

At the talk stage, students work with their groups using the Student Worksheet. Student Worksheet contains training questions that must be done by students in groups. The importance of talk in a learning is to be able to build understanding and knowledge together through interactions and conversations between individuals in the group. Finally able to provide solutions to the problems faced that boil down to an agreement in formulating the learning objectives to be achieved (Shoimin,2016: 213).*Write*

At this stage, the students write down the results of the discussions obtained from the first and second stage activities. According to Huda (2015: 219), the writing consists of the basis of the concept used, the association with the previous material, the settlement strategy, and the solutions obtained. Meanwhile, according to Shield and Swinson in Yamin (2012: 87) states that writing in mathematics helps realize one of the objectives of learning, namely the learner's understanding of the material he learns. Additionally Masingila and Wisniowska in Yamin (2012:88) suggests that students' writing activities for teachers can monitor students' mistakes, misconceptions, and learner conceptions of the same idea. As Cahyani and Hodijah stated in Zulkarnaini (2011: 145) that: writing skills are the most complicated skills because writing is not just copying words and sentences, but also developing and expressing thoughts in an orderly writing.

According to Elida (2012: 182) activities carried out by students during this stage are:

1. Write solutions to given problems including calculations
2. Doing the work step by step in a straight way
3. Check all work until it is believed that no one is left behind
4. Believe that the work done is complete, easy to read and guaranteed authenticity

To realize learning in accordance with the above expectations, the learning steps with the TTW strategy according to Yamin (2012:90) are as follows.

1. The teacher divides the reading text in the form of a student Activity Sheet containing open-ended problem situations and instructions and procedures for implementation.
2. Learners read the text and take notes from the reading results individually, to be taken to the discussion forum (Think).
3. Learners interact and collaborate with friends to discuss the content of notes (Talk). Teachers act as mediators of the learning environment.
4. Learners self-instruct knowledge as a result of collaboration (Write).

Based on the above learning steps, the implementation of TTW will be:

- 1) Learners read questions about understanding mathematical concepts in Student Worksheet and make important notes from individual reading results (think) for further discussion

2) Learners interact and collaborate with their friends to discuss the contents of notes and resolution of problems given in Student Worksheet (talk).

3) Learners write the results of a joint agreement on solving the problem of understanding mathematical concepts (write).

The final activity of learning is to make reflections and conclusions on the material studied. Before that, one or more students were selected as group representatives to present answers, while the other group was asked to provide a response

Grouping in think talk write type cooperative learning is done by heterogeneous grouping. Heterogeneous groups can be formed by paying attention to gender diversity, socioeconomic and ethnic backgrounds and academic abilities.

According to Lie (2002: 41) heterogeneous grouping measures based on academic ability are:

1. Sort learners by academic ability
2. Form the first group
3. Form the next group

The division of discussion groups to be conducted in this study is based on the academic ability of learners. Learners are sorted by their academic ability by looking at the results of daily maths replays so that they will see students with high, medium, and low academic abilities. Furthermore, a discussion group consisting of 3 to 4 people consisting of one high academically capable learner, one or two moderate academically capable learners, and one low academically capable learner.

Based on the description above, it can be concluded that the synth of cooperative learning activities think talk write type and its relation to cooperative learning steps are as follows:

Table 4. Syntax Cooperative Learning Activities Type Think Talk Write and Its Linkages To Cooperative Learning Steps

Phase	Participants' Behavior Educated	Related to cooperative learning
Phase 1 <i>Think</i>	Learners read texts and take notes from reading results individually (think), for the next Discussed	Present information for analysis by learners
Phase 2 <i>Talk</i> (Talk or discuss)	Learners interact and collaborate with groupmates to discuss the content of notes (talk). In this activity they use their own language and words to convey mathematical ideas in discussions. Understanding built through interactions in discussions, therefore discussions are expected to can produce solutions to problems that given	Helping group work and learning so that learners better understand the solutions to the problems provided

Phase 3 <i>Write</i>	Learners self-construct knowledge containing understanding and communication mathematics in the form of writing (write)	Evaluation by writing the results obtained at the stage of thinking and speaking
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(Huda,2014 :220)

3. Understanding Mathematical Concepts

In KBBI, understanding means understanding exactly. A person is said to understand when a person understands what something is meant and is able to explain it again. While understanding is a translation of the term Understanding which is interpreted as absorption of the meaning of a material studied.

While the Concept is the basis for a higher mental process to formulate principles and generalizations. According to Suherman (2003:33) "Concepts are abstract ideas that allow us to group objects into examples and non-examples".

Understanding the concept is one of the abilities that are expected to be mastered by learners in the learning process. While understanding the concept of mathematics is the level of ability at which learners are able to understand concepts, situations, and facts known, and can explain using his own knowledge without changing the meaning of.

Understanding concepts in mathematics learning is very important because the concepts of mathematics are continuous. Understanding the concept of mathematics is a prerequisite for mastering the next material or concept. As Suherman (2003: 22) stated, "In mathematics there are prerequisite topics or concepts as the basis for understanding the next topic or concept". Learners understand a concept can be seen from the indicator of understanding the concept.

According to Regulation of the Minister of Education and Culture No. 59 of 2014, several indicators of achievement of concept understanding skills include:

1. Reassess the concept that has been studied
2. Classifying objects based on the requirements that make up the concept
3. Identify the properties of the operation or concept
4. Applying the concept logically
5. Provide examples or examples of cons (not examples) of concepts studied
6. Present concepts of various forms of mathematical representation (tables, graphs, diagrams, drawings, sketches, mathematical models or other means)
7. Linking various concepts in mathematical concepts as well as outside mathematics
8. Developing necessary requirements and or conditions is quite a concept.

In this study, researchers use seven indicators of understanding mathematical concept according to Permendikbud No. 59 Year 2014 because have a complete and clearer indicator than other experts.

4. Learning Using curriculum 2013

Based on the 2013 curriculum, learning in schools uses a scientific approach. The scientific approach is a mandatory approach to learning in schools, both elementary and secondary schools that used the 2013 kurikulum. In this section, it will take a scientific approach, because the school that is the object of research has used the 2013 curriculum.

The approach of mathematics learning according to Suherman (2003: 6) is the way that teachers take the implementation of learning so that the concepts presented can be adapted by learners. While the scientific approach is closely related to scientific methods. Scientific methods (scientific) generally involve observation or observation activities needed for the formulation of hypotheses or collect data adjusted to the knowledge to be studied (Sani, 2014: 45)

Learning with scientific attachment is a learning process designed in such a way that learners actively formulate concepts, laws or principles through the stages of observing (to identify or find problems), formulate problems to propose and formulate hypotheses, collect data with various techniques, analyze data, draw conclusions and communicate concepts, laws or principles that are "found" (Daryanto, 2014:53)

According to Sani (2014:53) the application of scientific approaches in learning sees process skills such as observing, questioning, questioning, trying or gathering information, reasoning or association and communicating. This is in accordance with the learning process in the curriculum 2013 in Permendikbud number 58 year 2014 (2014: 53) confirmed that the learning process consists of five main learning experiences namely *observing* , *questioning* , gathering information (*experimenting*), reasoning or association (*associating*), and communicating

a. Observing

Learning activities carried out in the process of observing are reading, listening, listening, seeing and etc..

b. Questioning

Activities in the process of learning to ask questions, question and answer and discuss information that is not understood from what is observed or questions to get additional information about what is observed.

c. Collecting information (*experimenting*)

Learning activities carried out in the process of collecting information are activities by exploring, trying, discrete, demonstrating, imitating shapes or motions, conducting experiments, reading sources other than textbooks, collecting data from sources through questionnaires, interviews and modifying.

d. Reasoning or association (*associating*)

Learning activities carried out in the process of reasoning or association is an activity carried out by collecting, analyzing data in the form of categories, associating or connecting phenomena / related information in order to find a pattern and conclude.

e. Communicate

Learning activities carried out in the process of communicating are learning activities carried out by presenting reports from observations into the form of charts, diagrams, or graphs. Prepare a report in writing and present a report that includes processes, results, and conclusions orally

5. The Linkage of *Think Talk Write* Models with Scientific Approach to Understanding Mathematical Concepts

Think Talk Write demands and requires learners to formulate their own concepts of the material to be mastered through individual thinking, discussing with groupmates and classics, and then writing down the conclusions of individually studied concepts. This goes hand in hand with the *scientific* approach that the *saintific* approach requires learners to actively construct knowledge, propose and formulate hypotheses, collect data with various techniques, draw conclusions and communicate formulated concepts, laws or principles.

Think talk write model is done with several stages or learning steps, namely *think*, *talk*, *write*. At the *think* stage students are asked to read the *Student Worksheet* then make a note of what has been read. In creating or writing notes learners distinguish and unify the ideas obtained and then translate them into their own language. The next step is *talk*, where students are asked to communicate and provide input so as to improve the understanding of learners. The last stage is *write* that students are asked to be able to re-write the results of their thoughts after discussion.

The learning process of *Think Talk Write* with a *saintific* approach in the ability to understand concepts is related to each other. This can be seen in table 5 below:

Table 5. The Linkage of Think Talk Write with Scientific Approach in Understanding Mathematical Concepts and Learning Activities of Learners

Steps	Teacher Activities	Student Activities	Comprehension Indicators Concept
Step 1 <i>Think</i>	The teacher asks the problem/question	Students observe the problems given by the teacher and ask about the problems given (observe and ask)	Identify the properties of an operation or concept
	Teachers direct and motivate learners to identify and classify given problems	Learners organise and select information relevant in solving the problems given to the Student Worksheet (collecting information)	Identifying the properties of the operation or concept, Classifying objects based on the requirements that make up the concept, Associating various concepts in mathematical concepts as well as outside Math
Step 2 <i>Talk</i>	Teachers encourage learners to collect data/information needed to build ideas and motivate learners to choose strategies and approaches that are appropriate to solve the problem	Learners collect problem-related information from group discussions (gathering information and associating	Identify the properties of an operation or concept

Steps	Teacher Activities	Student Activities	Comprehension Indicators Concept
	The teacher asks the learner to presented its findings	Learners communicate problem solving that have been solved (communicate)	Reasate a concept that has been studied
	Teacher motivates other learners to respond to their friend's presentation	Learners respond to the results of group presentations and re-examine and prove the correctness of their findings (associating and communicating)	Presenting concepts from a variety of forms of mathematical representations (tables, graphs, diagrams, sketches, models mathematics or other means)
Step 3 <i>Write</i>	Teachers emphasize and direct learners to interpret the results of answers and draw conclusions	Learners write down conclusions and interpret the results of the answers that have been found (associate and communicate)	Presenting concepts from a variety of forms of mathematical representations (tables, graphs, diagrams, sketches, models mathematics or other means)

6. Direct Learning Models

The Direct Learning Model is a learning model that educators do directly in teaching basic skills and demonstrated directly to students with structured stages. The Direct Learning Model is expected to support the teaching and learning process for educators and learners, so that the expected learning objectives are achieved well and the learning outcomes obtained can improve well.

According to Regulation of the Minister of Education and Culture No. 59 (2014: 363) argues that the process of direct learning is an educational process in which learners develop knowledge, thinking skills and psychomotor skills through direct interaction with learning resources designed in the syllabus and *Lesson Plan* in the form of learning activities. In direct learning, students conduct learning activities observing, requesting, gathering information, associating or analyzing, and communicating what they have found in analytical activities..

According to Regulation of the Minister of Education and Culture No. 59 (2014 : 363), the characteristics of direct learning models include:Adanya tujuan pembelajaran dan prosedur penilaian hasil belajar.

- a. The overall syntax or pattern and flow of learning activities.
- b. Management system and learning environment that supports the ongoing and successful teaching.

The syntax of the direct learning model can be seen in Table 6 as follows:

Table 6. Syntax of Live Learning Activities

Phase	Phase	The Role of Educators
1	Delivering goals and preparing learners	Educators explain, teaching background information, the importance of lessons, preparing learners for learning.
2	Demonstrate knowledge and skills	Educators demonstrate skills correctly, or present information step by step.
3	Guiding training	Educators plan and provide initial training guidance.
4	Check understanding and provide feedback	Check if the learner has managed to do a good job, give feedback.
5	Provide opportunities for advanced training and implementation	Educators prepare for the opportunity to conduct advanced training, with particular attention to the application of more complex situations and daily life.

B. Relevant Research

Research relevant to the research that researchers will do refers to national and international journals. The following national and international journals related to research namely Fitriyana, Nur & Rani Asnurida (2018), showed that the influence of learning with *Think Talk Write* strategy on the results of mathematics learning, the contribution of learning interest to the results of learning mathematics and the interaction between learning strategies *Think Talk Write*.

This learning activity with *Think Talk Write* strategy habitues students to think first so that the learner can build his own year in understanding a mathematical concept. The difference of this research is the study using 5 indicators of understanding the concept, while the researchers used 8 indicators of understanding the concept according to permendikbud.

Suherman et al (2018), showed that there are significant differences in math learning outcomes between learners who are taught with *Think Talk Write* strategies and learners who are taught with conventional learning models. This *Think Talk Write* learning model actively involves students with the opportunity to discuss with group members. So that learners can share their ideas or ideas with their friends and from the results of their discussions get the right answers. The difference of this research is in the variables tied to the research conducted variables tied to the understanding of mathematical communication of learners.

Intan et al (2018), showed that understanding the mathematical concepts of learners who follow learning with *Think Talk Write* strategies is better than understanding the mathematical concepts of learners who follow conventional learning. Where the posttest experiment class is better than the control class which means there is an influence of *Think Talk Write* strategy on concept comprehension ability in mathematics learning. The difference of this research is the study using 4 indicators of understanding the concept, while the researchers used 8 indicators of understanding the concept according to permendikbud.

Sugandi, Asep Ikin (2011), showed that overall the initial level of mathematics ability factor of learners, Cooperative learning Type *Think Talk Write* and conventional learning provide a meaningful role to the achievement of communication skills and mathematical reasoning. However, the role of TTW learning is superior compared to the role of other factors in the communication ability and mathematical reasoning of learners. In addition, it was also concluded that there is an interaction between learning and the level of early mathematical ability of learners to the reasoning and mathematical communication skills of learners. The difference of this research is in the variables tied to the research that is carried out variables tied to the understanding of communication and mathematical reasoning of learners.

Sugandi, Asep Ikin (2011), showed that learners who are taught with *Think Talk Write* strategies and learners who are taught with conventional learning models play a meaningful role in the learning achievements of learners. The difference of this research is in the variables tied to the research conducted variables tied to understanding problem solving and mathematical connections of learners.

Setiawan, Eval & Indriwati, Sri Endah (2018), showed that there are significant differences in math learning outcomes between learners who are taught with *Think Talk Write* strategies and learners who are taught with conventional learning models. The difference of this research is in the variables tied to the research that is carried out the bound variable is lesson study.

Umami Khalimatus Sa'diyah et al (2019), showed that the use of *Think Talk Write* learning strategies is more effective than classes that use direct learning models. The difference from this research is in the variables tied to the research carried out variables terikannya understanding of mathematical communication of learners.

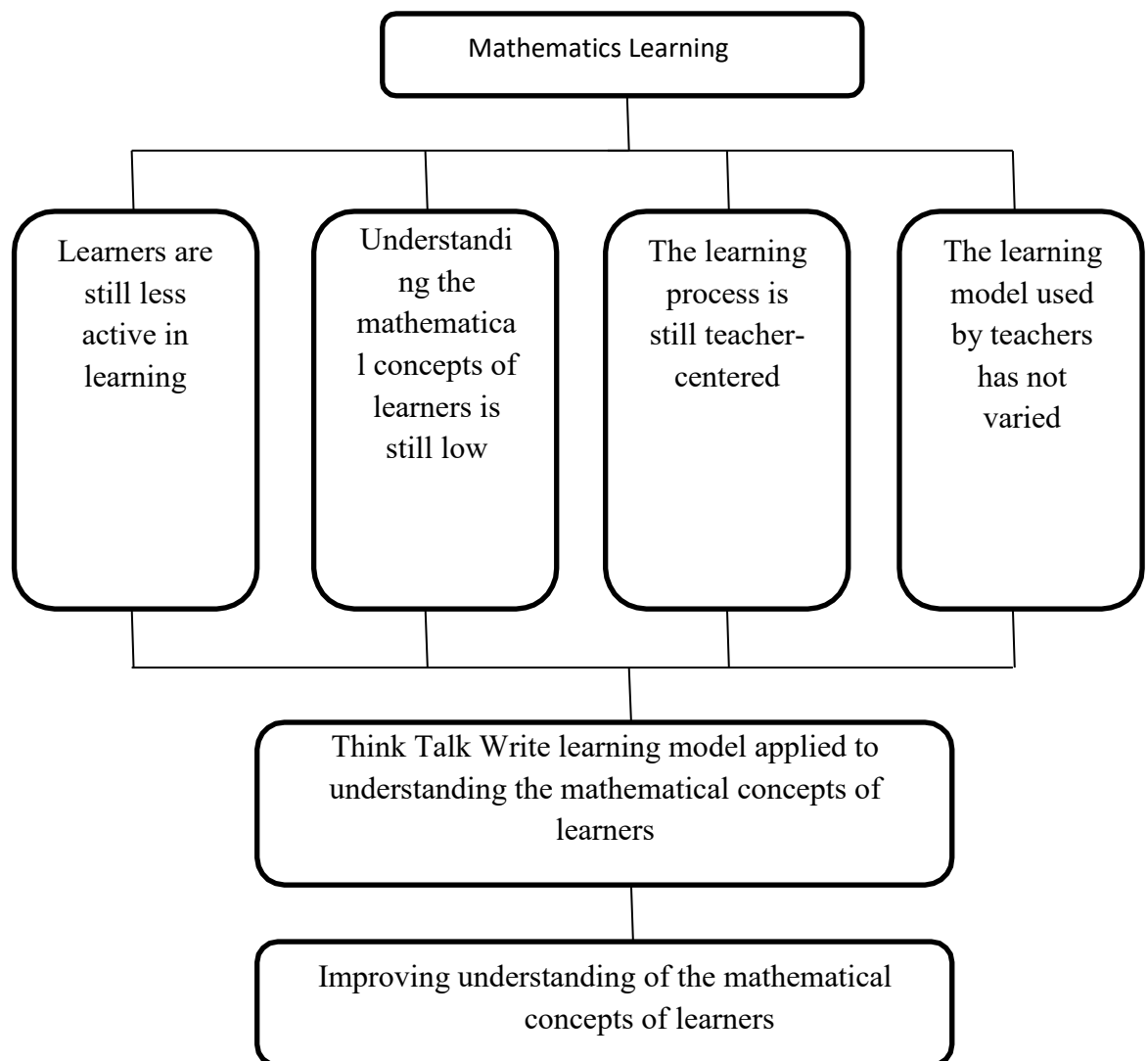
Ambarsari, Heny et al (2018), showed that the use of *Think Talk Write* learning strategies has a significant effect than classes that use direct learning models. The difference from this research is in the variables tied to the research conducted variables tied to the reading habits of learners.

Supandi et al (2018), showed that the use of *Think Talk Write* learning strategies has an effect on learners that students are eager to improve their mathematical abilities. The difference from this research is in the variables tied to the research conducted variables tied to the ability of representation of learners.

Rahmi Hidayati et al (2018), showed that there is an influence of learning with *Think Talk Write* strategy on mathematics learning results. The difference from this research is in the variables tied to the research conducted variables tied to the concept and communication ability of learners

C. Conceptual Framework

The conceptual framework in this study can be described as follows



D. Hypothesis

Based on theoretical studies, analysis of the linkages of think talk write learning models with indicators of understanding concepts and relevant research presented, the hypothesis in this study is the understanding of mathematical concepts of learners whose learning using the *Think Talk Write* learning model is better than understanding the mathematical concepts of learners whose learning uses direct learning at Adabiah 2 Padang High School.

CHAPTER III RESEARCH METHODS

A. Type and Design of Research

In accordance with the problems studied and the research objectives expressed, then this type of research is a four-year experimental research. The study was used to look at understanding the concepts of sample class learners. In this research design, two sample classes were selected, namely experimental class and control class. In the experimental class, learning is used using the *Think Talk Write* learning model, while in the control class the direct learning model is used. At the end of the study, tests will be conducted in both sample classes to see the understanding of the mathematical concepts of learners in both classes. The research design used is *Randomized Control Group Only Design* which can be seen in table 7 below:

Table 7. *Randomized Control Group Only Design* Research Design

Group	Treatment	Posttest
Experiment	X	T
Control	-	T

Source: Suryabrata (2004: 104)

Description:

X : The treatment given to the experimental class is the application of a cooperative model type *Think Talk Write*

T : Math concept comprehension test given in experiment class and control class

B. Population and Sample

The population in this study were students of grade XI MIA SMA Adabiah 2 Padang in the 2019/2020 school year. Here are the number of students in grade XI MIA SMA Adabiah 2 Padang Year Lesson 2019/2020.

Table 8. Students of Class XI MIA SMA Adabiah 2 Padang Lesson Year 2019/2020

No	Class	Learners in a class
1	XI MIA 1	23
2	XI MIA 2	26
3	XI MIA 3	36
4	XI MIA 4	35
Total		

Source: Adabiah 2 Padang High School Administration

1. Sample

Based on the problems to be examined, it takes two sample classes, one experimental class and one control class. The sampling steps are as follows:

- a. Collecting data on Final Semester Assessment of mathematics subjects of students in grade XI MIA SMA Adabiah 2 Padang in the 2019/2020 school year (cf. Appendix 1).
- b. Perform a flat similarity test

The average similarity test aims to find out if the population has an average similarity or not. In each test conducted using the help of *minitab software*. Before testing the similarity of the average population, the assumption test is carried out, namely normality test and population variance homogeneity test as follows:

1) Perform normality test

The normality test aims to see if the population is distributing normally or not. The test used was the *Anderson-Darling* test. The hypotheses tested are:

H_0 : Normal distributed population

H_1 : Population is not normally distribut

Distributed Statistical test formula is:

$$AD = \sum_{i=1}^n \frac{1-2i}{n} \{ \ln(F[z_i]) + \ln(1 - F[z_{n+1-i}]) \} - n$$

Where $z_i = \frac{x_i - \mu}{\sigma}$

$$CV = \frac{0.752}{1 + \frac{0.75}{n} + \frac{2.25}{n^2}}$$

Description:

AD : Anderson-Darling

F : Compulsive distribution function of special distribution

z_i : i-th standard number

n : sum of data

x_i : i-data

CV : Critical value

The decision rejected H_0 if $AD > CV$. After the test, it is carried out interpretation of the results obtained by the interpretation of P-value. Data criteria are normally distributed if the P-value obtained is greater than the real level () specified (= 0.05) or means accepted.

If on the contrary, the population does not distribute normally or is rejected. P-the value of each class can be seen in table 9 below.

Table 9. Population Member Normality Test Results

Class	P-value	Description
XI MIA 1	0,561	Normal distributed data
XI MIA 2	0,339	Normal distributed data
XI MIA 3	0,117	Normal distributed data
XI MIA 4	0,418	Normal distributed data

Based on the normality test, P-value > for each class. It can be concluded that a normal distributed population or H0 is accepted (cf. Appendix 2).

2) Perform variance homogeneity test

Variance homogeneity test aims to find out if the population has homogeneous variance or not. Homogeneity test is done with Bartlett test. The hypotheses on this homogeneity test are:

$$H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2$$

H_1 : there are at least two variances that are not the same

Calculation is done using minitab software. From the results of the calculation obtained, the interpretation of P-value with population criteria has homogeneous variance, if P-value > real level (α) = 0,05 or means H_0 diterima. Jika sebaliknya, maka variansi populasi tidak homogen atau H_0 rejected. Based on variance homogeneity test results obtained P-value =0,216.Means $P - value > \alpha$

so it can be concluded that the population has homogeneous variance or accepted H0 (cf. Appendix 3).

Because the data distributed normally and has a homogeneous variance, the average similarity test is carried out using parametric statistics that is a one-way *analysis of variance* (ANOVA) test.

Hypotheses in the average similarity test are as follows:

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_k$$

H_1 : at least two average values are not the same.

In this study, the average similarity test was done with the help of minitab software through the interpretation of P-value with (the ~~real~~ level set is 0.05). Data is stated to have similarities average or receive H0 if P-value $> \alpha$. Based on results average similarity obtained P-value = 0.787. It means P-value $> \alpha$ thus it can be concluded that the population has an average similarity or H0 is accepted (cf. Appendix 4).

c. Sampling class

Because the population has an average similarity, random sampling is carried out with the draw. The draw is done by taking a roll of paper that reads class XI MIA 1 to XI MIA 4. The class from the first take becomes the experimental group, while the class from the second take becomes the control group. The experimental class in this study was XI MIA 1 and the control class was XI MIA 2.

C. Variable and Data

1. Variable

The variable in this study consisted of 2 kinds of free variables and bound variables. The free variables in this study are mathematics learning by applying the Think Talk Write learning model to experimental classes and mathematics learning by applying a direct learning model to the control class. The bound variable is the understanding of the mathematical concept of students in grade XI of Adabiah 2 Padang High School.

2. Data

Research data consists of primary data and secondary data.

a) Primary data

Primary data is data obtained directly from the research subject. The primary data in this study is data on understanding the mathematical concepts of learners obtained from the test results at the end of the study. Primary data is data obtained directly from the research subject. The primary data in this study is data on understanding the mathematical concepts of learners obtained from the test results at the end of the study.

b) Secondary data

Secondary data in this study that became secondary data is data from the past math scores of students in grade XI sma Adabiah 2.

D. Research Procedure

The research that has been done is divided into three stages, namely the preparation stage, the implementation stage, and the completion stage.

1. Preparation Stage

- a. Determine the place and schedule of research and subject matter to be examined.

- b. Define sample classes that are experimental classes and control classes.
- c. Preparing learning tools, such as the Lesson Plan and the Student Worksheet (cf. Appendix 6, Appendix 8).
- d. Validating Lesson Plane and Student Worksheet. The learning device was validated by lecturers of Mathematics Department of FMIPA UNP, namely Mrs. Dra. Hj. Fitrani Dwina, M.Ed and Mr. Dr. Yerizon, M.Si (cf. Appendix 5, Appendix 7).
- e. Create grids, questions, answers and rubrics for mathematical concept comprehension tests (cf. Appendix 12, Appendix 13, Appendix 14).
- f. Validate the final test question to see if it is valid and feasible to use. The problem was validated by two mathematics lecturers (cf. Appendix 15).

2. Implementation Stage

a. Learning Process

In this study, the implementation of learning was carried out six times and at the next meeting was carried out a mathematical concept comprehension test. The research schedule can be seen in the following table.

Table 10. Research Schedule

No.					Teaching Materials
	Date (JP or Jam Pelajaran)	Experiment Class	Date (Jam Pelajaran)	Control Class	
1	17 February 2019(2 JP)	✓	17 February 2019(2 JP)	✓	Equation of circular tangents centered at point O(0,0) and radius r, if known gradients
2	18 February 2019(2 JP)	✓	20 February 2019(2 JP)	✓	Equation of circular tangents centered on point P(a,b) and radius r, if known gradients
3	24 February 2019(2 JP)	✓	24 February 2019(2 JP)	✓	Equation of the circle tangent line at point A (x_1, y_1) On the circle $x^2 + y^2 = r^2$
4	25 February 2019(2 JP)	✓	27 February 2019(2 JP)	✓	Equation of the circle tangent line at point A (x_1, y_1) On thr circle $(x - a)^2 + (y - b)^2 = r^2$
5	2 March 2019(2 JP)	✓	2 March 2019(2 JP)	✓	Equation of the circle tangent line at point A (x_1, y_1) On the circle $x^2 + y^2 + Ax + By + C = 0$
6	3 March 2019(2 JP)	✓	5 March 2019(2 JP)	✓	Equation of a circle tangent line at a point outside Circle
7	16 March 2019(2 JP)	✓	16 March 2019(2 JP)	✓	Final Test

The learning process in both sample classes is done with different models. In the experiment class using the *Think Talk Write* learning model while in the control class using direct learning. The learning steps implemented in the research can be explained as follows.

Table 11. Implementation Stage in Experiment and Control Class

Experiment Class	Control Class
<p>Preliminary activities</p> <ol style="list-style-type: none"> 1. The teacher greets and checks the presence of the students. 2. Teachers check the cleanliness, neatness, and readiness of learners to follow the learning. 3. The teacher asks the class president to lead a prayer to open the lesson <p>Phase 1. Delivering goals and motivating learner</p> <ol style="list-style-type: none"> 4. Teachers provide aperception and motivation. 5. The teacher conveys the objectives that the learners will achieve 6. Teachers motivate learners to learn and give an idea of the benefits of the material learned 7. The teacher conveys to the learner that the learning model to be used is the TTW learning model 	<p>a. Preliminary activities</p> <ol style="list-style-type: none"> 1) The educator asks one of the learners to lead the prayer. <p>Phase 1. Delivering Goals and preparing learners.</p> <ol style="list-style-type: none"> 2) Educators prepare students both physical and psychic in order to follow the learning process well (including checking the presence of learners). 3) Educators develop the motivation and interest in learning mathematics by showing the benefits of learning materials in their daily lives. 4) Educators convey learning objectives that will be achieved by learners. 5) As an adopter the educator associates the material to be studied with phenomena around the learner that can be observed and investigated by the learner.
<p>Core activities</p> <p>Phase 2. Presenting information</p> <ol style="list-style-type: none"> 8. The teacher presents the learning materials to be studied <p>Phase 3. Organizing learners into cooperative groups</p> <ol style="list-style-type: none"> 9. The teacher instructs the learner to sit in his/her group <p>Phase 4. Guiding the study group</p> <ol style="list-style-type: none"> 10. Teachers give StudentWorksheet to students <ul style="list-style-type: none"> • <i>Think stage</i> 	<p>b. Core activities</p> <p>Phase 2. Demonstrating Knowledge And Skills</p> <p>Educators deliver learning materials by demonstrating in front of the class.</p> <ol style="list-style-type: none"> a. A. Learners observe what educators demonstrate (OBSERVING) b. Students are allowed to ask questions if there is something they want to ask (QUESTIONING)

<p>Teachers direct individual learners to observe, read and understand issues, as well as record ideas in their own language</p> <ul style="list-style-type: none"> • <i>Talk stage</i> Teachers direct individual learners to observe, read and understand issues, as well as record ideas in their own language • <i>Write stage</i> Learners try to write the results of a joint agreement on solving the problem of understanding mathematical concepts <p>Phase 5. Evaluation</p> <ol style="list-style-type: none"> 11. The teacher appointed one of the groups to present the results of the discussion in the future of the class. 12. With the question and answer, the teacher directs all learners to the conclusion of the material learned at the meeting. <p>Phase 6. Awarding</p> <ol style="list-style-type: none"> 13. Teachers give award to the group. 	<p>Phase 3. Provide guided exercises</p> <ol style="list-style-type: none"> d. Pendidik Educators give examples of questions to learners and guide learners to understand the examples (tries) <p>Phase 4. Check comprehension and give slingshots back</p> <ol style="list-style-type: none"> d. Educators check students' understanding by giving exercises. e. Once done, the educator asks the uncomprehensible part and asks questions that require understanding of the concept to the learner. (Reasoning) f. If there is something to discuss, then the educator will discuss it on the board by asking one of the students to do it or the educator himself or herself who immediately explains.
<p>Closing Activities</p> <ol style="list-style-type: none"> 1. Teachers and students review learning materials. 2. The teacher and the learner conclude the learning. 3. The teacher conducts a quiz to test the learner's understanding of the material that has been understood 4. Learners are given tasks to do at home 	<p>Closing activities</p> <ol style="list-style-type: none"> 1) Educators and learners conclude the material that has been studied (communicate) 2) Reflection of the material that has been studied <p>Tahap Stage 5. Provide opportunities for advanced training and implementation</p> <ol style="list-style-type: none"> 3) Educators provide tasks for students to do at home.

5. The teacher ends the learning activity and tells the learner to read the next meeting material 6. The teacher closes the lesson by saying hamdalah.	4) Educator informs learners to learn the material for the next meeting. 5) Educators end Learning.
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b. Test Questions Mathematical Concept Understanding Test

Before the question was used in the sample class, the question test was conducted at Adabiah Padang High School on March 4, 2020. This is because Adabiah Padang High School has a close ranking based on average math scores, located in an area close to SMA Adabiah 2 Padang, using the same curriculum that is the 2013 curriculum, and has the same accreditation that is accreditation A. After that, an analysis of the test results of the question and the conclusion that the question is declared feasible. The average math score of grade XI of Adabiah Padang High School is 46.15 which can be seen in Appendix 16.

3. Completion Stage

The things that are done at the completion stage of the research are as follows.

- a. Provides a final test of mathematical concept understanding in both classes, both experimental and control classes after the subject has been studied.
- b. Process data from experiment classes and control classes. The test results of both classes of samples were analyzed.
- c. Find conclusions from the results obtained in accordance with the analysis used.

E. Research Instruments

In accordance with the problems in this research, the instruments used are:

1. Quizzes, Student Worksheet and Exercises

Quizzes, Student Worksheet and exercises were given each time during the study which was used to see the impact of *Think Talk Write* learning models on understanding the mathematical concepts of learners. This is measured through the development of understanding the concept of learners through the questions given at the end of the lesson.

2. Concept Comprehension Test

This concept comprehension test is used to compare the understanding of the mathematical concepts of learners. The test is in the form of an essay test according to the subject matter during the implementation of the *Think Talk Write* learning model which is compiled based on indicators of understanding mathematical concepts. This mathematical concept comprehension test instrument is used at the end of learning. To get a good test, the following steps are carried out:

1. Determine the purpose of conducting tests that is to find out the understanding of the mathematical concepts of learners
2. Analyze Kompetensi Dasar (KD).
3. Create restrictions on the material to be tested.

4. Create a grid of final test questions based on indicators of understanding mathematical concepts.
5. Create test question items based on the grid and the answer key.
6. The test question was validated by 2 lecturers of the Department of Mathematics, Padang State University (cf. Appendix 15)
7. Carry out test trials

The test test is done first before it is given to the sample class. Trials were conducted in classes in other schools that had similar characteristics to sample class characteristics.

8. Analyze test results

This analysis is done to identify the questions used, revised or discarded. To find out the quality of the test it is necessary to note the following:

a) The Differentiating Power Of The Problem

The differentiating power of the question is the ability to distinguish highly capable learners with low-skilled learners. Steps to determine the differentiating power of the problem is by looking for the problem differentiating index, namely:

- a. Data from the test results of understanding the mathematical concept of learners are sorted from the highest to the lowest grades.
- b. Then taken 27% of the number of students belonging to the high group and 27% of the number of students belonging to the low group.

- c. Calculate *degrees of freedom* (df) with

formula : $df = n_1 + n_2 - 2$

Description :

$df = \text{degrees of freedom}$

n_1 = many members of the high group

n_2 = many members of the low group

- d. Cari Search index differentiating the question with the formula stated Prawironegoro (1985: 11)

$$I_p = \frac{M_t - M_r}{\sqrt{\frac{\sum x_t^2 + \sum x_r^2}{n(n-1)}}}$$

Description :

I_p = question differentiating index

M_t = average high group score

M_r = average low group score

$\sum x_t^2$ = sum of squares of high group score deviation

$\sum x_r^2$ = sum of squares of low group score deviation

n = $27\% \times N$

N = many test taker

Suatu A question has a significant (significant) differentiating index if $I_{p \text{ hitung}} \geq I_{p \text{ tabel}}$ determined df (Prawironegoro, 1985: 11).

Based on the calculation results at a real level of 0.05 and $df = 10$ obtained the value of ip table = 2.23. The results of the calculation of the differentiating index of the test test can be seen in the following Table 12.

Table 12. Results of Calculation Of Differentiating Index Problem
TrialTabel 12. Test Match Index Calculation Results

Question number	Calculate I_p value	Differentiating Index Criteria
1	2,86	Significant
2	5,25	Significant
3	4,34	Significant
4	3,95	Significant
5	2,5	Significant
6	3,62	Significant
7	2,33	Significant

Based on Table 12 obtained I_p count greater than I_p table. So it can be concluded that the test question has a significant differentiating power. The calculation of the differentiating index of the full test test question can be found in Appendix 19.

b) Difficulty Index

A good problem is a question that is not too easy and not too difficult. To determine the difficulty index (I_k) used the formula stated by Prawironegoro (1985: 14):

$$I_k = \frac{Dt + Dr}{2mn} \times 100\%$$

Description :

I_k = difficulty index

Dt = number of high group scores

Dr = Sum of low group scores

m = Score each question if correct

n = 27% x N where N= sum of learners

Difficulty index criteria questions are :

a) the problem is stated very difficult, if $I_k = 0\%$

- b) The problem is declared difficult, if $0\% < I_k < 27\%$
- c) The problem is stated medium, if $27\% \leq I_k \leq 73\%$
- d) The problem is stated easy, if $73\% < I_k < 100\%$
- e) The problem is stated very easily, if $I_k = 100\%$

The results of the calculation of the difficulty index of the test test can be seen in Table 13.

Table 13. Test Test Difficulty Index Calculation Results

Question number	Value of I_k	Question criteria
1	55,55 %	Medium
2	89,58 %	Easy
3	83,33 %	Easy
4	70,83 %	Medium
5	25 %	Difficult
6	27,08 %	Medium
7	29,16 %	Medium

Based on Table 13 obtained 2 problems with easy categories, 4 problems with medium categories and 1 problem with difficult categories. The calculation of the difficulty index of the test test can be seen in Appendix 20.

c) Criteria for Acceptance of Questions

The classification of questions/items according to Prawironegoro (1985: 16) is:

- (1) The item remains in use if the I_p is significant and $0\% < I_k < 100\%$
- (2) Items are corrected if:

I_p significant and $I_k = 0\%$ or 100% I_p
 insignificant and $0\% < I_k < 100\%$

- (3) The item is replaced if the I_p is insignificant and $I_k = 0\%$ or $I_k = 100\%$

The results of the classification of acceptance of test questions can be seen in the

following Table 14.

Table 14. Test Acceptance Classification Results

Question number	Differentiating Index		Difficulty index		Classification
	I_p	Question criteria	I_k	Question criteria	
1	2,86	Significant	55,55 %	Medium	Used
2	5,25	Significant	89,58 %	Easy	Used
3	4,34	Significant	83,33 %	Easy	Used
4	3,95	Significant	70,83 %	Medium	Used
5	2,5	Significant	25 %	Difficult	Used
6	3,62	Significant	27,08 %	Medium	Used
7	2,33	Significant	29,16 %	Medium	Used

Based on Table 14 it is obtained that all test questions can be used on mathematical concept comprehension tests (cf. Appendix 21). Menghitung Reliabilitas Tes

Reliability test is done to get the level of accuracy (reliability or keajegan) data collection tool used. To determine the coefficient of reliability used alpha formulas expressed by Arikunto (2001: 109-110) namely:

$$r_{11} = \left[\frac{n}{n-1} \right] \left[1 - \frac{\sum \sigma_b^2}{\sigma_t^2} \right], \text{ with : } \sigma_t^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{N}}{N}$$

description :

r_{11} : reliability of the question

n : sum of the question

$\sum \sigma_b^2$: sum of variance scores each question

σ_t^2 : total variance

$\sum X$: the sum of scores of each problem

$\sum X^2$: sum of squares of each question's score

The criteria for reliability level referenced are:

$r_{11} = 1,00$: perfect reliability

$0.80 \leq r_{11} < 1.00$: very high reliability

$0.60 \leq r_{11} < 0.80$: high reliability

$0.40 \leq r_{11} < 0.60$: reliability is enough

$0.20 \leq r_{11} < 0.40$: low reliability

$0.00 < r_{11} < 0.20$: very low reliability

$r_{11} = 0.00$: no reliability

Reliability calculations about test tests provide $r_{11} = 0.60075$. This means that the test tested has high reliability. The reliability calculation of the test test can be seen in Appendix 22.

3. Develop a mathematical concept comprehension test.

F. Data Analysis Techniques

Analysis of mathematical concept comprehension test data aims to test whether the proposed hypothesis is accepted or rejected. Before conducting the hypothesis test, normality and variance homogeneity of both samples were carried out. The learner's answer to the mathematical concept comprehension test was also assessed based on a score on the rubric of mathematical concept understanding and question weight assessment. Data analysis is carried out in accordance with the following steps.

- a. Normality Test

The normality test aims to determine whether the sample class data is distributed normally or not. The test used was the Anderson Darling test. The hypotheses proposed are:

H_0 : Math concept comprehension score of normal distributed sample class learners

H_1 : The math concept comprehension score of the sample class learners was not normally distributed.

To see if the data is distributed normally or not, done by interpretation of P-Value (significant value) that is normal distributed data if the P-Value obtained is greater than the real level of $\alpha = 0.05$, P-value of each sample class can be seen in the following Table 15.

Table 15. Sample Class Normality Test Results

Class	P-value	Description
Experiment	0,863	Normal distributed data
Control	0,085	Normal distributed data

Based on the normality test, P-value $> \alpha$ for each sample class. It can be concluded that normal distributed or H0 test data is received. Normality test results can be seen in Appendix 25.

b. Variance Homogeneity Test


Homogeneity tests were performed on final test data aimed at finding out if both samples had homogeneous variances or not. This test can be done using an F-test. The hypothesis according to Walpole (1992: 314) in homogeneity testing is as follows.

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Description :

σ_1^2 : Variance of the value of understanding the concept of a group of students whose learning uses the Think Talk Write learning model

: Variance of the value of understanding the concept of a group of students whose learning uses a direct learning model

In this study, variance homogeneity test using the help of minitab *software* with interpretation comparing P-value with α (the specified real level is 0.05). Data declared homogeneous or H_0 received if P-value is $> \alpha$ the results of variance homogeneity test P-value = 0.911. This means that P-value $> \alpha$ so that it can be concluded that the sample class test data has homogeneous variance or H_0 is accepted. The results of variance homogeneity test can be seen in Appendix 26.

After conducting normality and homogeneity test obtained, obtained normal and homogeneous distributed data, then conducted hypothesis test using t-test.


c. Hypothesis Test


The hypothesis test was conducted to determine the mathematical concept understanding of the learners of the experimental class better than the understanding of the mathematical concept of the control class learners. Based on the hypothesis presented, a one-party test with statistical hypothesis test is:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 > \mu_2$$

Description :

 : average score of final test results understanding the mathematical concept of the group learners using the Think Talk Write learning model.

 : average score of final test results understanding the mathematical concept of the group learners using a direct learning model.

After the hypothesis test using minitab software results with the criteria of reject H_0 test if $P\text{-value} > \alpha$ and in this study obtained $P\text{-value} = 0.003$. It means reject H_0 or average the results of the test understanding the mathematical concept of the learners of the experimental class are better than the control class. The results of the research hypothesis test can be seen in Appendix 27.

CHAPTER IV RESEARCH RESULTS AND DISCUSSION

A. Research result

The results of the study were obtained through the application of the *Think Talk Write* learning model to the understanding of students' mathematical concepts which were compared with students' understanding of mathematical concepts using the direct learning model at SMA Adabiah 2 Padang.

1. Description Data

Based on research that was conducted on February 17 to March 16 2020, there were two kinds of research data, namely data from quizzes, Student Worksheet and exercises to measure the development of students' mathematical conceptual understanding and final test results to measure the level of understanding of students' mathematical concepts. The following is a description of the two research data.

a. Quizzes, Student Worksheet and Exercises

The development of students' understanding of mathematical concepts in class XI MIA SMA Adabiah 2 Padang is seen based on the number of students who get the highest score for each meeting as long as the *Think Talk Write* learning model is applied. Meeting I was attended by 22 students, Meeting II was attended by 23 students, Meeting III was attended by 21 students, Meeting IV was attended by 22 students, Meeting V was attended by 23 students, Meeting VI was attended by 22 learners.

This is because 1 participant students did not attend school at meeting I, 2 students did not attend school at meeting III, 1 student did not attend school at meeting IV, and 1 student did not attend school at meeting VI.

The development of students' mathematical conceptual understanding can be seen from the number of students who get the highest score on each quiz as well as at each lkpd and exercise can be seen in Table 16.

Table 16. Percentage of Total Students with Maximum Score

Indicator	Meeting to-											
	1				2				3			
	K	L	T	%	K	L	T	%	K	L	T	%
1	-	-	9	40.9	-	-	-	-	-	-	14	66.6
2	0	4	-	9.09	1	4	-	21.7	-	-	-	-
3	-	-	-	-	-	-	-	-	3	10	-	30.9
4	-	-	-	-	-	-	6	26.0	-	7	-	33.3
5	-	-	4	18.1	-	-	-	-	-	-	7	30.4
7	-	-	-	-	-	-	-	-	-	-	8	38.0
8	-	-	-	-	-	-	-	-	-	-	-	-

Indicator	Meeting to-											
	4				5				6			
	K	L	T	%	K	L	T	%	K	L	T	%
1	-	-	-	-	-	-	-	-	-	-	17	77.2
2	-	-	6	27.2	-	-	8	34.7	-	-	-	-
3	4	11	-	34.0	8	-	-	34.7	-	-	-	-
4	-	9	-	40.9	-	11	-	47.8	-	-	-	-
5	-	-	-	-	-	-	-	-	-	-	10	45.4
7	-	-	-	-	-	-	10	43.4	-	-	-	-
8	-	-	5	22.7	-	-	5	34.7	10	11	13	51.5

Description:

Indicator 1 : Reassess the concept that has been studied

Indicator 2 : Classify objects based on the requirements that the concept meets

Indicator 3 : Identify the properties of the operation or concept

Indicator 4 : Applying the concept logically

Indicator 5 : Provides examples or examples of cons (not examples) of concepts studied

Indicator 7 : Linking various concepts in mathematical concepts as well as outside Mathematics

Indicator 8 : Developing necessary terms and or conditions is quite a concept

K : Many students get the maximum score in the quiz

L : Many learners get the maximum score in Student Worksheet

Q : Many learners get the maximum score in training

% : The percentage of the number of learners who get the maximum score

Based on table 16 can be seen that the percentage of the number of learners who maximum score on each indicator in six meetings increased.

b. Mathematical Concept Comprehension Test

Comparison of mathematical concept understanding of learners who learn using think talk write learning model (experimental class) with mathematical concept understanding of learners who learn using direct learning model (control class) seen from the results of mathematical concept comprehension test in the form of description. The test was conducted at the end of the study on March 16, 2020. The test was attended by 23 students and the control class was also attended by 26 students. The test result data can be seen in Table 17.

Table 17. Results of the Sample Class Mathematical Concept Understanding Test Results

Class	total Learner s	Average - Average Score	Standard Deviation	Highest Score Score	Lowest Score Score
Experiment	23	20.26	2.68	26	15
Control	26	17.85	2.75	22	12

In Table 17, it can be seen that the average test score of the experimental class is higher than the control class. The average score for the experimental class was 20.26, while the average test score for the control class was 17.85 from a maximum total score of 27. The highest score in the experimental and control class was 26 and 22. This means that the highest score in the experimental class is higher than the control class. The lowest scores in the experimental and control classes are 15 and 12. This means that the lowest score in the experimental class is higher than the control class. However, the standard deviation of the experimental class is lower than the control class. The standard deviation of the experimental class is 2.68 and the standard deviation of the control class is 2.75. This shows that the understanding of the mathematical concepts of control class students is more diverse than the experimental class. Complete calculations can be seen in attachment 23 and attachment 24.

More detailed sample class student test data can be seen through each item of test questions in accordance with the indicators of understanding the mathematical concept under study. The ability of students on each indicator is scored according to the rubric of the assessment of the mathematical concept understanding test.

The results have expressed in terms of numbers and percentages in the experimental class and control class can be seen in Table 18.

Table 18. Percentage of students in the sample class who received a score of 0 - 4 on the Mathematical Concept Understanding Ability Test

Class	Indicator	No. Question	Score 4		Score 3		Score 2		Score 1		Score 0	
			F	%	F	%	F	%	F	%	F	%
E	1	1	-	-	19	82.60	3	13.04	1	4.34	0	0.00
K			-	-	15	57.69	5	19.23	6	23.07	0	0.00
E	2	2	20	86.95	3	13.04	0	0.00	0	0.00	0	0.00
K			16	61.53	4	15.38	6	23.07	0	0.00	0	0.00
E	3	3	10	43.47	10	43.47	3	13.04	0	0.00	0	0.00
K			1	3.84	15	57.69	7	26.92	0	0.00	3	11.53
E	4	4	4	17.39	16	69.56	3	13.04	0	0.00	0	0.00
K			4	15.38	14	53.84	4	15.38	4	15.38	0	0.00
E	5	5	3	13.04	7	30.43	11	47.82	2	8.69	0	0.00
K			0	0.00	13	50.00	12	46.15	1	3.84	0	0.00
E	7	6	2	8.69	8	34.78	7	30.43	4	17.39	2	8.69
K			0	0.00	16	61.53	8	30.76	0	0.00	2	7.69
E	8	7	4	17.39	11	47.82	5	21.73	1	4.34	2	8.69
K			0	0.00	16	61.53	3	11.53	0	0.00	7	26.92
Average Percentage Score experimental class				26.70		45.95		19.87		4.96		2.48
Mean Percentage Score of control class				11.53		51.09		24.72		6.04		6.59

Information:

- Indicator 1 : Restate concepts that have been studied
- Indicator 2 : Classify objects based on fulfilled requirements for which the concept is
- Indicator 3 : Identifies the properties of the operation or concept
- Indicator 4 : Apply the concept logical
- Indicator 5 : Provide examples or cons (not examples) of that concept studied
- Indicator 7 : Link various concepts in mathematical concepts as well outside mathematics
- Indicator 8 : Develop a necessary and / or sufficient condition concept
- E : Class experiment
- K : Class control
- F : Frequency / number of participants students each score

Overall, Table 18 explains that the average student in the experimental class who obtained a score of 4 was higher than the control class with a percentage of the experimental class of 26.70% while the control class was 11.53%. Meanwhile, the average student in the experimental class who obtained scores of 3.2 and 1 was less than the control class.

Overall it was concluded that the understanding of the experimental class students 'mathematical concepts was better than the control class students' mathematical conceptual understanding. This fact supports that the results of the hypothesis testing are reject H_0 . The results of the hypothesis show that the understanding of the mathematical concepts of students who learn using the *Think Talk Write* learning model is better than the understanding of the mathematical concepts of students who learn using conventional learning in class XI MIA SMA Adabiah 2 Padang (refer to Appendix 27).

2. Data analysis

a. Data analysis

The development of students' mathematical conceptual understanding can be seen based on:

- 1) The percentage of students who get the maximum score on quizzes, Student Worksheet, and exercises on each indicator of understanding mathematical concepts

The percentage of the number of students who obtained the maximum score on quizzes, Student Worksheet, and exercises can be seen in Table 16. Based on Table 16, it can be seen that the percentage of students who obtained the maximum score on each indicator in six meetings has increased.

Indicator I is restating the concepts that have been learned. The percentage of students who got the maximum score on Indicator I was 40.9% at the first meeting, 66.6% at the third meeting, and 77.2% at the sixth meeting. It can be concluded that in indicator I there was an increase in the percentage of students who got the maximum score in three meetings.

Indicator II is to classify objects based on whether or not the requirements that make up the concept are met. The percentage of students who got the maximum score on Indicator II was 9.09% at the first meeting, 21.7% at the second meeting, 27.2% at the fourth meeting, and 34.7% at the fifth meeting. It can be concluded that in indicator II there is an increase in the percentage of students who get the maximum score in four meetings.

Indicator III is to identify the characteristics of the operation or mathematical concept. The percentage of students who got the maximum score on Indicator III was 30.9% at the third meeting, 34.0% at the fourth meeting, and 34.7% at the fifth meeting. It can be concluded that in indicator III there is an increase in the percentage of students who get the maximum score in three meetings.

Indicator IV is applying the concept logically. The percentage of students who got the maximum score on the IV indicator was 26.0% at the second meeting, 33.3% at the third meeting, 40.9% at the fourth meeting, and 47.8% at the fifth meeting. It can be concluded that in indicator IV there is an increase in the percentage of students who get the maximum score in four meetings.

Indicator V is to provide examples or cons (not examples) of the concepts being studied. The percentage of students who got the maximum score on indicator V was 18.1% at the first meeting, 30.4% at the third meeting and 45.4% at the sixth meeting. It can be concluded that in indicator V there is an increase in the percentage of students who get the maximum score in three meetings.

Indicator VII is linking various concepts in mathematical concepts and outside mathematics. The percentage of students who got the maximum score on the VII indicator was 38.0% at the third meeting and 43.4% at the fifth meeting. It can be concluded that in indicator VII there was an increase in the percentage of students who got the maximum score in two meetings.

Indicator VIII is developing the necessary and / or sufficient conditions for a concept. The percentage of students who got the maximum score on indicator VII was 22.7% at the fourth meeting, 34.7% at the fifth meeting and 51.5% at the sixth meeting. It can be concluded that in indicator VIII there was an increase in the percentage of students who got the maximum score in three meetings.

So it can be concluded that in general the percentage of students who get the maximum score on quizzes, Student Worksheet, and exercises on each indicator of understanding mathematical concepts has increased.

b. Data Analysis of Mathematical Concept Understanding Ability Tests

Data analysis in this study aims to test statistically whether the hypothesis being tested is accepted or rejected. The hypothesis of this study is that the understanding of students 'mathematical concepts using the *Think Talk Write* learning model is better than students' understanding of mathematical concepts using direct learning. Before testing the hypothesis, first the normality test is carried out and followed by the variance homogeneity test with the help of Minitab *software*.

1) Normality test

The normality test used the *Anderson-Darling* test. In the experimental class obtained a *P-value* of 0.863 and in the control class a *P-value* of 0.085 was obtained. Because the *P-value* in the experimental class and control class is more than the real level, $\alpha = 0.05$, it can be concluded that the test data of the two sample classes are normal distributed. The results of the normality test for the second class of the sample class there is an attachment (refer to Appendix 25).

2) Homogeneity Test of Variance

Homogeneity test using the F-test. Based on the calculation results obtained a P-*value* of 0.911. Because the P-*value* is more than the real level, $\alpha = 0.05$, it can be concluded that the data has homogeneous variance. The results of the sample class variance homogeneity test are in the appendix (refer to Appendix 26).

3) Hypothesis testing

Research hypothesis testing aims to determine whether the proposed research hypothesis is accepted or rejected. Based on the results of the analysis, it was obtained that the experimental class test data and the control class test data were normally distributed and homogeneous, then hypothesis testing was carried out using the *t-test* which can be seen in the attachment (refer to Appendix 27). Based on the hypothesis test, the P-*value* is 0.003. This shows that the P-*value* is less than the real level of 0.05, so reject H_0 or accept H_1 . This means that the average value of the students' mathematical concept understanding test in the experimental class is better than the average value of the class students' mathematics concept understanding test control.

The following describes the analysis of the influence data on each indicator of understanding mathematical concepts using the *Think Talk Write* learning model in the experimental class and the direct learning model in the control class.

a) Indicator 1

Indicator 1 of understanding mathematical concepts is restating the concepts that have been learned. This indicator is represented by question number 1. The question number 1 is as follows.

Explain what is meant by tangents to circles!

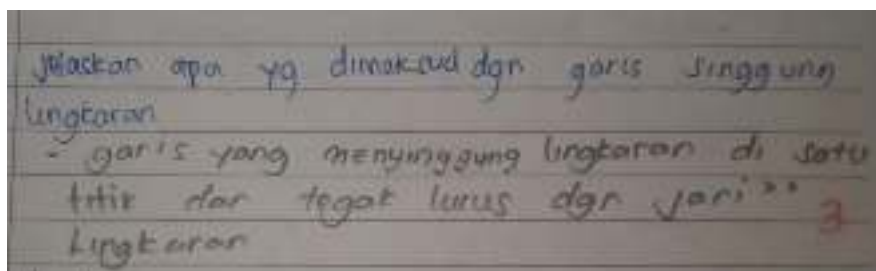
In this question, students are expected to be able to meet the indicators of restating the concepts that have been learned. Based on the questions presented, students can explain again the understanding of the material tangent to the circle. The percentage of students in the experimental class and control class who received a score of 0 - 3 for question number 1 is seen in the following table.

Table 19. Percentage of students in the experimental class and control class who received a score of 0 - 3 for Problem Number 1

Class	The percentage score for Problem Number 1			
	0	1	2	3
Experiment	0.00	4.34	13.04	82.62
Control	0.00	23.07	19.23	57.70

Based on the table, it can be seen that the scores obtained by the sample class students are given based on the scoring rubric of the mathematical concept understanding test where the highest score is 3 and the lowest score is 0. It is clear that the percentage of the experimental class students' ability scores in restating

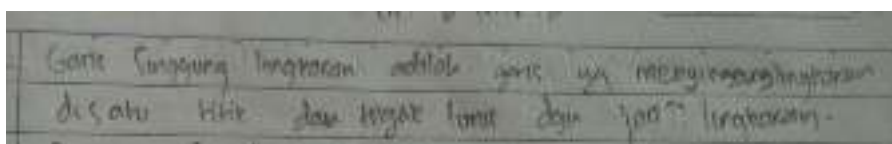
concepts that have been learned better than control class students. Examples of answers to students who get a score of 3 are as follows.



Question: explain what is a circle tangent

Answer: the line that offends the circle at one point and is perpendicular to the radius of the circle

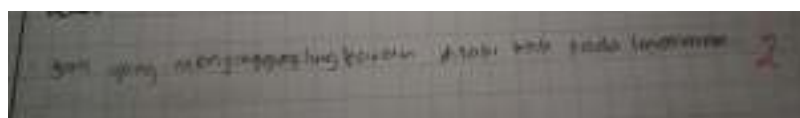
Figure 5. Examples of answers to experimental class students who received a score of 3 in question number 1



Answer : a circular tangent is a garis that combines one point and is perpendicular to the radius of a circle

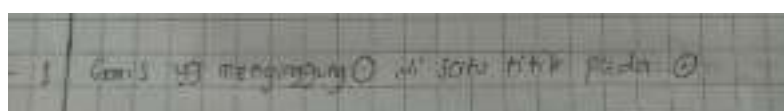
Figure 6. Examples of answers to control class students who get a score of 3 in question number 1

In Figures 5 and 6 it can be seen that the answers of students are able to restate the concepts that have been studied in accordance with the rubric so that they get a score of 3. In addition to getting a score of 3, there are students who get a score of 2. The following are examples of students' answers to the experimental class and control class get a score of 2.



Answer : line that alludes to a circle or point on a circle

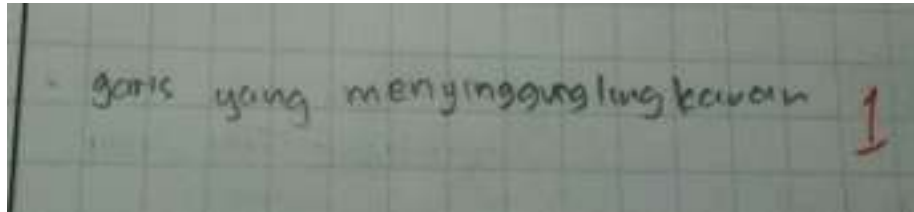
Figure 7. Examples of answers to experimental class students who get a score of 2 in question number 1



Answer : line that offends the circle at one point on the circle

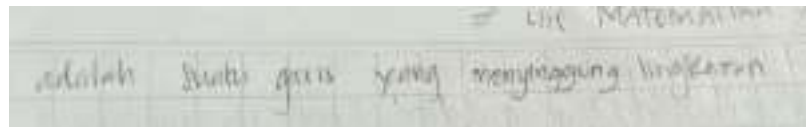
Figure 8. Examples of answers to control class students who get a score of 2 in question number 1

In Figures 7 and 8 it can be seen that students have restated the concepts that have been learned and have provided explanations, but the explanations given are not complete. Therefore, students' answers like this get a score of 2. Furthermore, there are examples of answers to students who get a score of 1.



Answer : line that offends the circle

Figure 9. Examples of answers to experimental class students who get a score of 1 in question number 1



Answer : is a garis that offends the circle

Figure 10. Examples of answers to control class students who get a score of 1 in question number 1

In Figures 9 and 10 it can be seen that students have restated the concepts they have learned. The answers are correct but not complete. So that the answers of students like this get a score of 1.

b) Indicator 2

Indicator 2 of understanding mathematical concepts is to classify objects based on the requirements that make up the concept. This indicator is represented by question number 2. The question number 2 is as follows.

Find the equation tangent to circle

$$x^2 + y^2 = 20 \text{ and graded 2!}$$

In this question, students are expected to be able to meet the indicators of classifying objects based on the requirements of the concept. This problem requires students to determine the equation tangent to the circle $x^2 + y^2 = 20$ and graded 2. In this case students can use the equation tangent to the circle if the gradient is known. The percentage of students in the experimental class and control class who get a score of 0- 4 for question number 2 can be seen in Table 20 below.

Table 20. Percentage of students in the experimental class and control class who received a score of 0 - 4 for Problem Number 2

Class	The percentage score on question number 2				
	0	1	2	3	4
Experiment	0.00	0.00	0.00	13.04	86.96
Control	0.00	0.00	23.07	15.38	61.55

Based on Table 20, it can be seen that the percentage of students in the experimental class in classifying objects based on fulfilled the concept requirements who get a score of 4 is 86.96% of students. While the control class who got a score of 4 was 61.55% of the students in that class. Examples of answers to students who get a score of 4 are as follows.

$r = \sqrt{20}$
 $m = 2$
 $y = mx \pm r \sqrt{1+m^2}$
 $= 2x \pm \sqrt{20} \sqrt{1+2^2}$
 $= 2x \pm \sqrt{20} \sqrt{5}$
 $= 2x \pm \sqrt{100}$
 $= 2x \pm 10$
 Jadi, persamaannya adalah
 $y = 2x + 10$ dan
 $y = 2x - 10$

Figure 11. Examples of answers to experimental class students who got a score of 4 in question number 2

Diketahui:
 $x^2 + y^2 = 20$ gradien : 2 $r = \sqrt{20}$
 Jawab:
 $y = mx \pm r \sqrt{1+m^2}$
 $y = 2x \pm \sqrt{20} \sqrt{1+2^2}$
 $y = 2x \pm \sqrt{100} = \pm 10$
 $y = 2x \pm 10$
 Jadi, persamaan garis singgungannya
 $y = 2x + 10$ $y = 2x - 10$

Figure 12. Examples of answers to control class students who get a score of 4 in question number 2

In Figures 11 and 12 it can be seen that students have been able to classify objects based on fulfilling the requirements of the concept. Students have been able to classify the answers to question number 2

uses the equation for the tangent to the circle if the gradient is known exactly. For answers, students like this are given a score of 4. In addition, there are students who get a score of 3.

Handwritten student work for Figure 13:

Dik: $x^2 + y^2 = 20$ $m = 2$ $r = \sqrt{20}$

Dit: $y = ?$

Jawab: $y = mx \pm r \sqrt{1+m^2}$

$y = 2x \pm \sqrt{20} \cdot \sqrt{1+2^2}$

$y = 2x \pm \sqrt{20} \cdot \sqrt{5}$

$y = 2x \pm \sqrt{100}$

$y = 2x \pm 10$

The final answer is marked with a red '3'.

Figure 13. Examples of answers to experimental class students who got a score of 3 in question number 2

Handwritten student work for Figure 14:

Diketahui:

$x^2 + y^2 = 20$ $m = 2$

Dit: $y = ?$ $r = \sqrt{20}$

Jawab:

$y = mx \pm r \sqrt{1+m^2}$

$y = 2x \pm \sqrt{20} \cdot \sqrt{1+2^2}$

$y = 2x \pm \sqrt{20} \cdot \sqrt{5}$

$y = 2x \pm \sqrt{100}$

$y = 2x \pm 10$

Figure 14. Examples of answers to control class students who get a score of 3 in question number 2

On Picture 13 and 14 seen that participants students has been able classify objects based on the requirements met the concept

the. But did not make any conclusions from the results obtained. For answers like this, students are given a score of 3.

lingkaran

$$x^2 + y^2 = 20, m = 2$$

$$x^2 + y^2 = 20$$

Pusat $(0,0) \rightarrow r = \sqrt{20}$

$$m = 2$$

$$PG = \frac{|y - mx|}{\sqrt{1+m^2}}$$

2

Figure 15. An example of an answer to a control class student who got a score of 2 in question number 2

In Figure 15 it can be seen that students did not write down the continuation of the answer. Students only correctly write the equation tangent to the circle if the gradient is known. So that students do not get the correct answer. For answers like this students get a score of 2.

c) Indicator 3

Indicator 3 of understanding mathematical concepts is to identify the properties of the operation or concept. The ability of students to identify the properties of the operation or concept is tested in question number 3 as follows.

Check whether the equation is tangent to the circle $(x - 10)^2 + (y - 1)^2 = 61$ through point $(15, -5)$ ie $5x - 6y = 105$ by first identifying where the point is located on or outside circle!

On question this participants students expected able fulfill indicator identify the properties of the operation or concept. This problem requires students to prove the result of the equation tangent to the circle.

The percentage of students in the experimental class and control class who received a score of 0 - 4 for question number 3 is seen in Table 21 below.

Table 21. Percentage of students in the experimental class and control class who received a score of 0 - 4 for Problem Number 3

Class	The percentage of score on question number 3				
	0	1	2	3	4
Experiment	0.00	0.00	13.04	43.47	43.49
Control	11.53	0.00	26.92	57.69	3.86

Based on Table 21, it can be seen that the percentage of students in the experimental class in identifying the properties of the operation or concept who received a score of 4 was 43.49% of students. While the control class who got a score of 4 was 3.86% of students. This means that the ability of experimental class students in identifying the properties of operations or concepts is better than the control class. Examples of answers to students who get a score of 4 are as follows.

Handwritten student work for Figure 16:

$$(x-10)^2 + (y-1)^2 = 61$$

$$(15-10)^2 + (5-1)^2 = 61$$

$$5^2 + 4^2 = 61$$

$$25 + 16 = 41$$

$$41 \neq 61$$

Handwritten student work for Figure 17:

$$(x-10)^2 + (y-1)^2 = 61$$

Substitusi (15, 5) ke

$$(15-10)^2 + (5-1)^2 = 61$$

$$5^2 + 4^2 = 61$$

$$25 + 16 = 41$$

Hasilnya 41, bukan 61. Jadi, titik (15, 5) bukan merupakan titik pada lingkaran.

Figure 16. Examples of answers to experimental class students who received a score of 4 for question number 3

Handwritten student work for Figure 17:

$$(x-10)^2 + (y-1)^2 = 61$$

Substitusi (15, 5) ke

$$(15-10)^2 + (5-1)^2 = 61$$

$$5^2 + 4^2 = 61$$

$$25 + 16 = 41$$

Hasilnya 41, bukan 61. Jadi, titik (15, 5) bukan merupakan titik pada lingkaran.

Figure 17. Examples of answers to control class students who get a score of 4 for question number 3

In Figures 16 and 17 it can be seen that the answers of students have identified the properties of the operation or concept using the formula for the tangent to circle equation. So that students get a score of 4. In addition, there are students who get a score of 3 as follows.

$$3x + 4y = 10 \quad (1)$$

$$2x + 3y = 7 \quad (2)$$

$$(1) - (2) \Rightarrow (3x + 4y) - (2x + 3y) = 10 - 7$$

$$x + y = 3 \quad (3)$$

$$(3) \times 2 \Rightarrow 2x + 2y = 6 \quad (4)$$

$$(2) - (4) \Rightarrow (2x + 3y) - (2x + 2y) = 7 - 6$$

$$y = 1$$

$$(3) \times 3 \Rightarrow 3x + 3y = 9 \quad (5)$$

$$(1) - (5) \Rightarrow (3x + 4y) - (3x + 3y) = 10 - 9$$

$$y = 1$$

$$(3) \times 4 \Rightarrow 4x + 4y = 12 \quad (6)$$

$$(1) - (6) \Rightarrow (3x + 4y) - (4x + 4y) = 10 - 12$$

$$-x = -2 \Rightarrow x = 2$$

$$x = 2, y = 1$$

$$3(2) + 4(1) = 10$$

$$2(2) + 3(1) = 7$$

$$6 + 4 = 10$$

$$4 + 3 = 7$$

$$10 = 10$$

$$7 = 7$$

$$x = 2, y = 1$$

Figure 18. Examples of answers of experimental class students who get a score of 3 for question number 3

$$3x + 4y = 10 \quad (1)$$

$$2x + 3y = 7 \quad (2)$$

$$(1) - (2) \Rightarrow (3x + 4y) - (2x + 3y) = 10 - 7$$

$$x + y = 3 \quad (3)$$

$$(3) \times 2 \Rightarrow 2x + 2y = 6 \quad (4)$$

$$(2) - (4) \Rightarrow (2x + 3y) - (2x + 2y) = 7 - 6$$

$$y = 1$$

$$(3) \times 3 \Rightarrow 3x + 3y = 9 \quad (5)$$

$$(1) - (5) \Rightarrow (3x + 4y) - (3x + 3y) = 10 - 9$$

$$y = 1$$

$$(3) \times 4 \Rightarrow 4x + 4y = 12 \quad (6)$$

$$(1) - (6) \Rightarrow (3x + 4y) - (4x + 4y) = 10 - 12$$

$$-x = -2 \Rightarrow x = 2$$

$$x = 2, y = 1$$

$$3(2) + 4(1) = 10$$

$$2(2) + 3(1) = 7$$

$$6 + 4 = 10$$

$$4 + 3 = 7$$

$$10 = 10$$

$$7 = 7$$

$$x = 2, y = 1$$

Figure 19. Examples of answers to control class students who get a score of 3 for question number 3

In Figures 18 and 19 it can be seen that students have correctly identified the properties of the operation or concept. But did not make any conclusions from the results obtained. So that students with answers like this are given a score of 3. In addition, there are students who get a score of 2 as follows.

Figure 20. Examples of answers to experimental class students who get a score of 2 for question number 3

Figure 21. Examples of answers to control class students who get a score of 2 for question number 3

In Figures 20 and 21 it can be seen that the process of students has not determined the equation of the tangent to the circle but only determines the location of the point on the circle. So that students with answers like this are given a score of 2.

d) Indicator 4

Indicator 4 of understanding mathematical concepts is applying the concept logically. The ability of students to apply the concept logically was tested in question number 4 as follows.

Determine the equation for the tangent to the circle
 $x^2 + y^2 - 6x - 4y - 12 = 0$ which is perpendicular to the line $y = x + 4$!

In this question, students are expected to be able to meet the indicators of applying the concept logically. This problem requires students to determine the equation tangent to a circle that is perpendicular to a line. The percentage of students in the experimental class and control class who received a score of 0 - 4 for question number 4 is seen in Table 22 below.

Table 22. Percentage of Students in the Experiment Class and Control Class who obtained a score of 0 - 4 for Question Number 4

Class	The percentage score on question number 4				
	0	1	2	3	4
Experiment	0.00	0.00	13.04	69.56	17.40
Control	0.00	15.38	15.38	53.84	15.40

Based on Table 22, it can be seen that the percentage of students in the experimental class in applying the concept logically obtained a score of 4, namely 17.40% of students. While the control class who got a score of 4 was 15.40% of students. This means that the ability of experimental class students to apply the concept logically is better than the control class. Examples of answers to students who get a score of 4 are as follows.

$$L: x^2 + y^2 - 6x - 4y - 11 = 0$$

$$P: y = x + 1$$

Menentukan pusat dan jari-jari lingkaran:

$$x^2 + y^2 - 6x - 4y - 11 = 0$$

$$(x^2 - 6x) + (y^2 - 4y) = 11$$

$$(x^2 - 6x + 9) + (y^2 - 4y + 4) = 11 + 9 + 4$$

$$(x - 3)^2 + (y - 2)^2 = 24$$

Pusat: $(3, 2)$
 Jari-jari: $\sqrt{24} = 2\sqrt{6}$

Menentukan titik potong:

$$y = x + 1$$

$$(x - 3)^2 + (x + 1 - 2)^2 = 24$$

$$(x - 3)^2 + (x - 1)^2 = 24$$

$$x^2 - 6x + 9 + x^2 - 2x + 1 = 24$$

$$2x^2 - 8x - 14 = 0$$

$$x^2 - 4x - 7 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 28}}{2} = \frac{4 \pm \sqrt{44}}{2} = 2 \pm \sqrt{11}$$

Titik potong: $(2 + \sqrt{11}, 3 + \sqrt{11})$ dan $(2 - \sqrt{11}, 3 - \sqrt{11})$

Figure 22. Examples of answers to experimental class students who received a score of 4 for question number 4

$$L: x^2 + y^2 - 6x - 4y - 11 = 0$$

$$P: y = x + 1$$

Menentukan pusat dan jari-jari lingkaran:

$$x^2 + y^2 - 6x - 4y - 11 = 0$$

$$(x^2 - 6x) + (y^2 - 4y) = 11$$

$$(x^2 - 6x + 9) + (y^2 - 4y + 4) = 11 + 9 + 4$$

$$(x - 3)^2 + (y - 2)^2 = 24$$

Pusat: $(3, 2)$
 Jari-jari: $\sqrt{24} = 2\sqrt{6}$

Menentukan titik potong:

$$y = x + 1$$

$$(x - 3)^2 + (x + 1 - 2)^2 = 24$$

$$(x - 3)^2 + (x - 1)^2 = 24$$

$$x^2 - 6x + 9 + x^2 - 2x + 1 = 24$$

$$2x^2 - 8x - 14 = 0$$

$$x^2 - 4x - 7 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 28}}{2} = \frac{4 \pm \sqrt{44}}{2} = 2 \pm \sqrt{11}$$

Titik potong: $(2 + \sqrt{11}, 3 + \sqrt{11})$ dan $(2 - \sqrt{11}, 3 - \sqrt{11})$

Figure 23. Examples of answers to control class students who get a score of 4 for question number 4

In Figures 22 and 23, it can be seen that students have been able to determine the equation of a tangent to a circle that is perpendicular to a line and make conclusions from what is obtained. Answers of students like this get a score of 4.

In addition, there are students who get a score of 3 as follows.

$$x^2 + y^2 - 6x - 4y - 12 = 0$$

$$y = x + 4$$

$$P(5, 2)$$

$$r = \sqrt{(3-5)^2 + (2-2)^2}$$

$$= \sqrt{4 + 0}$$

$$= \sqrt{4}$$

$$= 2$$

$$m_1 = 1$$

$$m_1 \cdot m_2 = -1$$

$$m_2 = -1$$

$$(y - 2) = m(x - 5) \pm r \sqrt{1 + m^2}$$

$$(y - 2) = -1(x - 5) \pm 2 \sqrt{1 + (-1)^2}$$

$$(y - 2) = -x + 5 \pm 2\sqrt{2}$$

$$y = -x + 5 \pm 2\sqrt{2}$$

$$P(5, 2) \rightarrow -x + 5 + 2\sqrt{2}$$

$$y = -x + 5 - 2\sqrt{2}$$

Figure 24. Examples of answers to experimental class students who received a score of 3 for question number 4

$L: x^2 + y^2 - 6x - 4y - 12 = 0$ \Rightarrow center $(3, 2)$ radius $r = 5$
 Jwb:
 $P: \left(\frac{3}{2}, \frac{1}{2}\right)$
 $r = \sqrt{\left(\frac{3}{2} - 3\right)^2 + \left(\frac{1}{2} - 2\right)^2} = \sqrt{\frac{9}{4} + \frac{9}{4}} = \sqrt{\frac{18}{4}} = \frac{3\sqrt{2}}{2}$
 $r = 5$
 $m = 1$
 $(y - 2) = m(x - 3) \pm r \sqrt{1 + m^2}$
 $(y - 2) = 1(x - 3) \pm 5\sqrt{1 + 1}$
 $y - 2 = x - 3 \pm 5\sqrt{2}$
 $y = x + 2 \pm 5\sqrt{2}$
 $y = x + 2 + 5\sqrt{2}$
 $y = x + 2 - 5\sqrt{2}$
 Pers: $y = x + 2 + 5\sqrt{2}$
 $y = x + 2 - 5\sqrt{2}$

Figure 25. Examples of answers to control class students who get a score of 3 for question number 4

In Figures 24 and 25, it can be seen that students have been able to determine the equation of a tangent to a circle that is perpendicular to a line but did not write down the conclusions of the results obtained. Answers like this get a score of 3.

$L: x^2 + y^2 - 6x - 4y - 12 = 0$ \Rightarrow center $(3, 2)$ radius $r = 5$
 Jwb:
 $P: \left(\frac{3}{2}, \frac{1}{2}\right)$
 $r = \sqrt{\left(\frac{3}{2} - 3\right)^2 + \left(\frac{1}{2} - 2\right)^2} = \sqrt{\frac{9}{4} + \frac{9}{4}} = \sqrt{\frac{18}{4}} = \frac{3\sqrt{2}}{2}$
 $r = 5$
 $m = 1$
 $(y - 2) = m(x - 3) \pm r \sqrt{1 + m^2}$
 $(y - 2) = 1(x - 3) \pm 5\sqrt{1 + 1}$
 $y - 2 = x - 3 \pm 5\sqrt{2}$
 $y = x + 2 \pm 5\sqrt{2}$
 $y = x + 2 + 5\sqrt{2}$
 $y = x + 2 - 5\sqrt{2}$

Figure 26. Examples of answers of experimental class students who get a score of 2 for question number 4

$6: x^2 + y^2 - 6x - 4y - 12 = 0$
 lingkaran pusat garis $y = x + 4$
 \Rightarrow
 $P = (3, 2)$
 $r = \sqrt{3^2 + 2^2 + 12} = 5$
 $k = \frac{|1(3) - 1(2) + 4|}{\sqrt{1^2 + 1^2}} = 5$
 $\sqrt{2} = 5$
 $\sqrt{2} = 5$
 $1) - 5 = 4$
 $2) - 5 = 4$
 $(2-2) - 1(1-4) + 4 = 5$
 $(2-2) - 1(1-4) + 4 = 5$
 $4 = 5$
 $5 = x + 4$
 $x = 5 - 4 = 1$
 $5 = x + 4$
 $x = 5 - 4 = 1$
 $1 = x + 4$
 $x = 1 - 4 = -3$
 $1 = x + 4$
 $x = 1 - 4 = -3$

Figure 27. Examples of answers to control class students who get a score of 2 for question number 4

In Figures 26 and 27 it can be seen that students have not been able to find the equation tangent to the circle if it is known to be a line. Students answer only by looking for the radius and gradient of the line. That way, students get a score of 2. Furthermore, below are the answers of students with a score of 1.

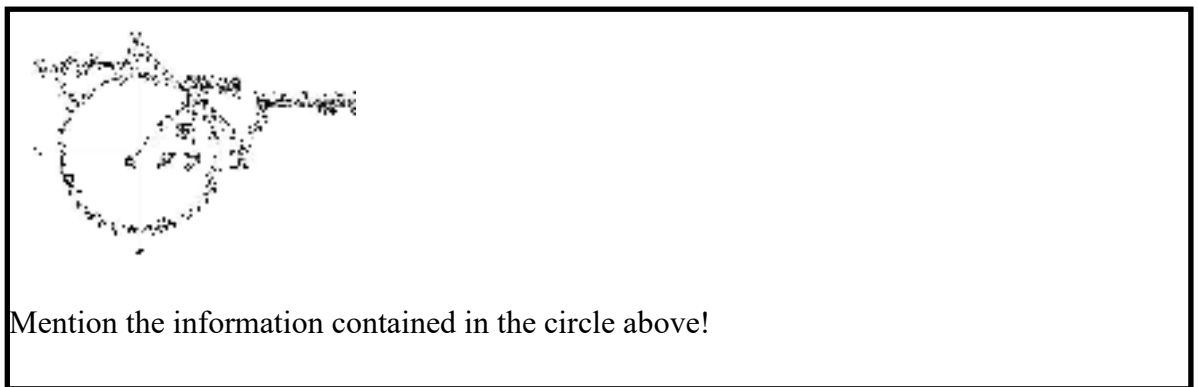
$01: x^2 + y^2 - 6x - 4y - 12 = 0$
 lingkaran pusat $y = x + 4$
 $P = (3, 2)$
 $r = 5$
 1

Figure 28. Examples of answers to control class students who get a score of 1 for question number 4

In Figure 28 it can be seen that students have not been able to apply the concept logically, students only write what is known in the questions so that students are given a score of 1.

e) Indicator 5

Indicator 5 is understanding mathematical concepts, which is to provide examples or not examples of the concepts being studied. The abilities of students are tested in question number 5 as follows.

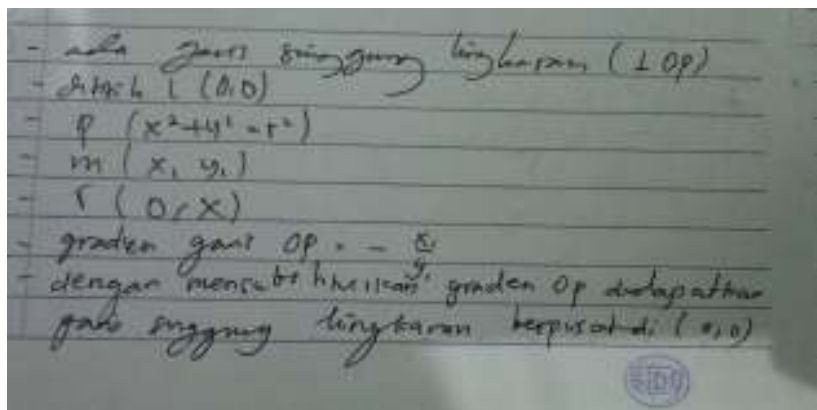


In this question, students are expected to be able to meet the indicators providing examples or not examples of the concepts being studied. The percentage of students in the experimental class and control class who got a score of 0 - 4 for question number 5 is seen in Table 23 below.

Table 23. Percentage of Students in the Experiment Class and Control Class who received a score of 0 - 4 for Question Number 5

Class	Percentage score on question number 5				
	0	1	2	3	4
Experiment	0.00	8.69	47.82	30.43	13.06
Control	0.00	3.84	46.15	50.01	0.00

Based on Table 23, it can be seen that the percentage of students in the experimental class in determining samples or not samples obtained a score of 4, namely as much as 13.06% of students. While the control class who got a score of 4 was 0.00% of students. This means that the ability of experimental class students to provide examples or not examples of the concepts learned is better than the control class. Examples of answers to students who get a score of 4 are as follows.



Answer :

- there is a circle tangent (to p)
- titik l(0,0)
- $p(x^2 + y^2 = r^2)$
- $m(x_1, y_1)$
- $r(0, x)$
- gradient line $op = -x_1/y_1$
- by substituting the op value obtained the tangent line of the centered circle (0,0)

Figure 29. Examples of answers to experimental class students who received a score of 4 for question number 5

Figure 29 shows that students have been able to provide the information contained in the circle. So that students get a score

4. In addition, there are students who get a score of 3 as follows.

$L.O = x^2 + y^2 = r^2$
 - titik $P(x_1, y_1)$
 - garis singgung $= x x_1 + y y_1 = r^2$ (tgp)
 - gradien garis $op = \frac{y_1}{x_1}$ dan $m_t = -\frac{x_1}{y_1}$

Answer :

- circle $x^2 + y^2 = r^2$
- point $p(x_1, y_1)$ tangent $= x x_1 + y y_1 = r^2$ (tgp)
- gradient of line $op = \frac{y_1}{x_1}$ and $m = -\frac{x_1}{y_1}$

Figure 30. Examples of answers to experimental class students who received a score of 3 for question number

- Pers. $O = x^2 + y^2 = r^2$
 - garis singgung tegak lurus OP
 - gradien garis $OP = \frac{y_1}{x_1}$ dan $m_t = -\frac{x_1}{y_1}$

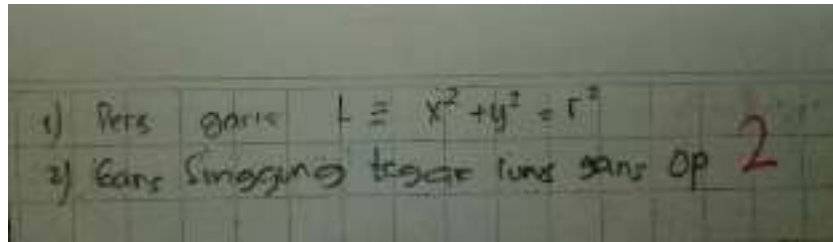
Answer :

- equation of the circle $x^2 + y^2 = r^2$
- tangent line perpendicular
- gradient line $op = \frac{y_1}{x_1}$ and $m = -\frac{x_1}{y_1}$

Figure 31. Examples of answers to control class students who get a score of 3 for question number 5

In Figures 30 and 31 it can be seen that students have been able to provide the information contained in the circle, but there is little information that is missing.

So that students get a score of 3

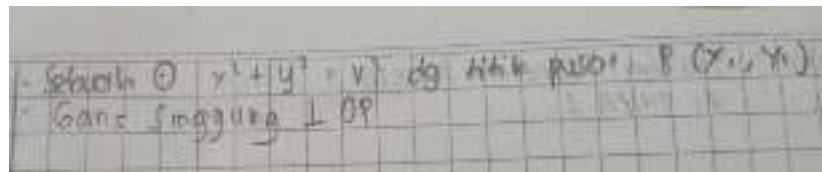


Answer :

- equation $x^2 + y^2 = r^2$

-tangent line perpendicular op

Figure 32. Examples of answers of experimental class students who get a score of 2 for question number 5



Answer :

- a circle $x^2 + y^2 = r^2$ with center point $p(x_1, y_1)$

-tangent line perpendicular op

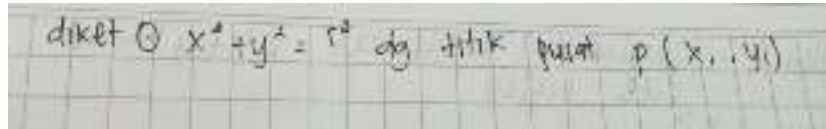
Figure 33. Examples of answers to control class students who get a score of 2 for question number 5

In Figures 32 and 33, it can be seen that students have answered the answers correctly, but the information provided is only a little. So that students get a score of 2.



Answer : circle equation $= x^2 + y^2 = r^2$

Figure 34. Examples of answers of experimental class students who get a score of 1 for question number 5



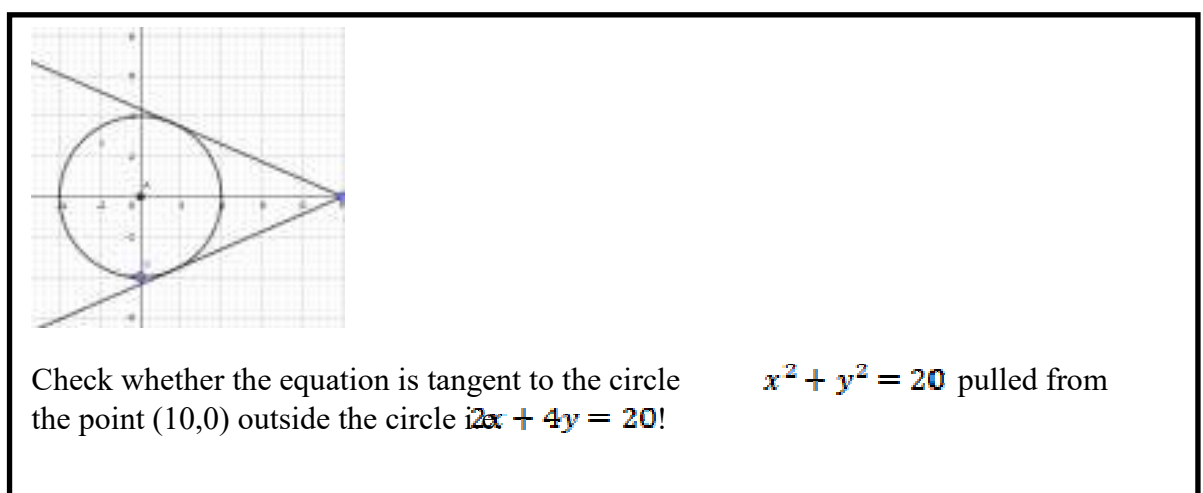
Answer : circle equation $x^2 + y^2 = r^2$ with center point $p(x_1, y_1)$

Figure 35. Examples of answers to control class students who get a score of 1 for question number 5

Figures 34 and 35 show that students have not been able to provide examples or not examples of the concepts being learned so that students are given a score of 1.

f) Indicator 7

The 7 indicator for understanding mathematical concepts is linking various concepts in mathematics and outside mathematics. The ability of students to relate various concepts in mathematics and outside mathematics is tested in question number 6 as follows.



In this problem, students are expected to be able to meet indicators linking various concepts in mathematics and outside mathematics. In this problem, students expect to determine the equation of the tangent to the circle drawn from a point outside the circle. The percentage of students in the experimental class and control class who got a score of 0 - 4 for question number 6 is seen in Table 24 below.

Table 24. Percentage of students in the experimental class and control class who received a score of 0 - 4 for Problem Number 6

Class	The percentage of score on question number 8				
	0	1	2	3	4
Experiment	8.69	17.39	30.43	34.78	8.71
Control	7.69	0.00	30.76	61.55	0.00

Based on Table 24 it can be seen that the percentage of students in the experimental class in linking various concepts in mathematics and outside mathematics who obtained a score of 4 was 8.71% of students. Meanwhile, there were no students in the control class who got a score of 4. This means that the ability of the experimental class students in linking various concepts in mathematics and outside mathematics is better than the control class. Examples of answers to students who get a score of 4 are as follows.

6. Metode: persamaan lingkaran $x^2 + y^2 = 20$
 titik $C(4, 2)$
 Misalkan
 Sumbu
 $y = x + b$
 $16 + 4 = 20$
 $20 = 20$
 $b = 0$
 $y = x$
 $x^2 + y^2 = 20$
 $x^2 + x^2 = 20$
 $2x^2 = 20$
 $x^2 = 10$
 $x = \sqrt{10}$
 $y = \sqrt{10}$
 $(x, y) = (\sqrt{10}, \sqrt{10})$
 $x_1 x_2 + y_1 y_2 = r^2$
 $2x + 2y = 20$
 Jadi persamaan tangen ke lingkaran adalah
 adalah $2x + 2y = 20$

Figure 36. Examples of answers to experimental class students who received a score of 4 for question number 6

In Figure 36, it can be seen that students have determined the equation tangent to the circle if it is known that a point is outside the circle so that students get a score of 4.

6. Metode: persamaan lingkaran $x^2 + y^2 = 20$
 titik $C(4, 2)$
 Misalkan
 Sumbu
 $y = x + b$
 $16 + 4 = 20$
 $20 = 20$
 $b = 0$
 $y = x$
 $x^2 + y^2 = 20$
 $x^2 + x^2 = 20$
 $2x^2 = 20$
 $x^2 = 10$
 $x = \sqrt{10}$
 $y = \sqrt{10}$
 $(x, y) = (\sqrt{10}, \sqrt{10})$
 $x_1 x_2 + y_1 y_2 = r^2$
 $2x + 2y = 20$

Figure 37. Examples of answers to experimental class students who received a score of 3 for question number 6

(6) Dik: $x^2 + y^2 = 20$
 Dit: y
 Jawab:

$x_1x + y_1y = r^2$ $10x + 0y = 20$ $10x = 20$ $x = \frac{20}{10}$ $x = 2$	$x^2 + y^2 = 20$ $2^2 + y^2 = 20$ $4 + y^2 = 20$ $y^2 = 20 - 4$ $y^2 = 16$ $y = \pm 4 \quad (2, 4)$
--	---

maka $x_1x + y_1y = r^2$
 $2x + 4y = 20$

Figure 38. Examples of answers to control class students who get a score of 3 for question number 6

In Figures 37 and 38, it can be seen that students have determined the equation of the tangent to the circle if it is known that a point is outside the circle but did not write down the conclusions of the results obtained so that students were given a score of 3.

6. Ditetahui persamaan lingkaran $x^2 + y^2 = 20$
 Ditanya: y ?
 Jawab:

$$x_1x + y_1y = r^2$$

$$10x + 0y = 20$$

$$10x = 20$$

$$x = \frac{20}{10}$$

$$x = 2$$

Figure 39. Examples of answers of experimental class students who get a score of 2 for question number 6

Diket : $x^2 + y^2 = 20$
 Dik : $y = ?$
 Jawab :
 $x, x + y, y = ?$
 $10x + 0y = 20$
 $10x = 20$
 $x = \frac{20}{10}$
 $x = 2$

Figure 40. Examples of answers of control class students who get a score of 2 for question number 6

In Figures 39 and 40 it can be seen that students have tried to answer the questions given but there are still ways that are lacking so that students are given a score of 2.

6. Diket PGSV $= x^2 + y^2 = 20$
 titik $(10, 0)$

Figure 41. Answers of students who get a score of 1 for question number 6

In Figure 41, it can be seen that students only write down what is known in the questions so that students are given a score of 1.

g) Indicator 8

Indicator 8 of understanding mathematical concepts is to develop the necessary or sufficient conditions of a concept. The ability of students in

develop the necessary or sufficient requirements of a concept tested in question number 7 as follows.

Circle $x^2 + y^2 - 2x + 13y + 40 = 0$ intersects the Y axis at $y = -5$.
 Check whether the equation of the tangent is $3y - 2x + 15 = 0$!

In this question, students are expected to be able to meet the indicators of developing the necessary or sufficient requirements of a concept. This problem requires students to check the equation of tangents to a circle if a circle intersects the Y-axis at a point by taking into account the necessary or sufficient conditions of a concept. The percentage of students in the experimental class and control class who got a score of 0 - 4 for question number 7 is seen in Table 25 below.

Table 25. Percentage of students in the experimental class and control class who received a score of 0 - 4 for Problem Number 7

Class	Percentage score on question number 6				
	0	1	2	3	4
Experiment	8.69	4.34	21.73	47.82	17.42
Control	26.92	0.00	11.53	61.55	0.00

Based on Table 25, it can be seen that the percentage of students in the experimental class in developing the necessary or sufficient requirements of a concept who get a score of 4 is 17.42% of students. While the control class who got a score of 4 was 0.00% of students. This means the ability of experimental class students in developing conditions

need or sufficient condition of a concept is better than control class. Examples of answers to students who get a score of 4 are as follows.

Handwritten mathematical work for Figure 42. The work is divided into two parts, labeled 'Jawab' and 'Jawab'.

Part 1 (Jawab):

$$\begin{aligned}
 & 3x + 4y = 2x + 13y + 40 \quad | -x| \\
 & x = 13y + 40 \quad | -13y| \\
 & 2x = 40 \quad | :2| \\
 & x = 20
 \end{aligned}$$

Part 2 (Jawab):

$$\begin{aligned}
 & 3x + 4y = 2x + 13y + 40 \quad | -x| \\
 & x = 13y + 40 \quad | \cdot \frac{1}{2}| \\
 & \frac{1}{2}x = \frac{13}{2}y + 20 \quad | \cdot 2| \\
 & x = 13y + 40
 \end{aligned}$$

Substituting $x = 13y + 40$ into the second equation:

$$\begin{aligned}
 & 3(13y + 40) + 4y = 2(13y + 40) + 13y + 40 \\
 & 39y + 120 + 4y = 26y + 80 + 13y + 40 \\
 & 39y + 120 + 4y = 39y + 120 \\
 & 39y + 120 = 39y + 120
 \end{aligned}$$

Final result: $x = 11/10$

Figure 42. Examples of answers of experimental class students who obtained score 4 for question number 7

Handwritten mathematical work for Figure 43. The work is divided into two parts, labeled 'Jawab' and 'Jawab'.

Part 1 (Jawab):

$$\begin{aligned}
 & 3x + 4y = 2x + 13y + 40 \quad | -x| \\
 & x = 13y + 40 \quad | \cdot \frac{1}{2}| \\
 & \frac{1}{2}x = \frac{13}{2}y + 20 \quad | \cdot 2| \\
 & x = 13y + 40
 \end{aligned}$$

Part 2 (Jawab):

$$\begin{aligned}
 & 3x + 4y = 2x + 13y + 40 \quad | -x| \\
 & x = 13y + 40 \quad | \cdot \frac{1}{2}| \\
 & \frac{1}{2}x = \frac{13}{2}y + 20 \quad | \cdot 2| \\
 & x = 13y + 40
 \end{aligned}$$

Substituting $x = 13y + 40$ into the second equation:

$$\begin{aligned}
 & 3(13y + 40) + 4y = 2(13y + 40) + 13y + 40 \\
 & 39y + 120 + 4y = 26y + 80 + 13y + 40 \\
 & 39y + 120 + 4y = 39y + 120 \\
 & 39y + 120 = 39y + 120
 \end{aligned}$$

Final result: $x = 11/10$

Figure 43. Contih the answers of control class students who get a score of 4 for question number 7

In Figures 42 and 43, it can be seen that students have been able to check the equation of tangents to a circle if a circle crosses the Y axis at a point and provides conclusions from the results obtained. So that students get a score of 4. In addition to getting a score of 4, there are students who get a score of 3 as follows.

$$\text{Diketahui: } x^2 + y^2 = 25 \text{ dan } (3, 4) \text{ pada lingkaran}$$

$$\text{Jawab: } x^2 + y^2 = 25$$

$$C = (0, 0)$$

$$r = \sqrt{25} = 5$$

$$x^2 + y^2 = 25$$

$$(3, 4) \text{ pada lingkaran}$$

$$3x + 4y = 25$$

$$\text{Diketahui: } x^2 + y^2 = 10 \text{ dan } (1, 3) \text{ pada lingkaran}$$

$$\text{Jawab: } x^2 + y^2 = 10$$

$$C = (0, 0)$$

$$r = \sqrt{10}$$

$$x^2 + y^2 = 10$$

$$(1, 3) \text{ pada lingkaran}$$

$$x + 3y = 10$$

3

Figure 44. Examples of answers to experimental class students who received a score of 3 for question number 7

Handwritten mathematical work for Figure 45:

$$x^2 + y^2 - 2x + 13y + 40 = 0 \quad \text{, memotong} \quad y = -x$$

$$x^2 + (-x)^2 - 2x + 13(-x) + 40 = 0$$

$$x^2 + x^2 - 2x - 13x + 40 = 0$$

$$2x^2 - 15x + 40 = 0$$

$$x = \frac{15 \pm \sqrt{15^2 - 4 \cdot 2 \cdot 40}}{2 \cdot 2}$$

$$x = \frac{15 \pm \sqrt{225 - 320}}{4}$$

$$x = \frac{15 \pm \sqrt{-95}}{4}$$

$$x = \frac{15 \pm i\sqrt{95}}{4}$$

Then, substituting back into $y = -x$:

$$y = -\left(\frac{15 \pm i\sqrt{95}}{4}\right) = \frac{-15 \mp i\sqrt{95}}{4}$$

So the points of tangency are:

$$\left(\frac{15 + i\sqrt{95}}{4}, \frac{-15 - i\sqrt{95}}{4}\right) \quad \text{and} \quad \left(\frac{15 - i\sqrt{95}}{4}, \frac{-15 + i\sqrt{95}}{4}\right)$$

Using the point-slope form of a line:

$$y - y_1 = m(x - x_1)$$

$$y - \frac{-15 - i\sqrt{95}}{4} = -1 \left(x - \frac{15 + i\sqrt{95}}{4}\right)$$

$$y + \frac{15 + i\sqrt{95}}{4} = -x + \frac{15 - i\sqrt{95}}{4}$$

$$y + x = \frac{15 - i\sqrt{95}}{4} - \frac{15 + i\sqrt{95}}{4}$$

$$y + x = \frac{15 - i\sqrt{95} - 15 - i\sqrt{95}}{4}$$

$$y + x = \frac{-2i\sqrt{95}}{4}$$

$$y + x = \frac{-i\sqrt{95}}{2}$$

$$x + y = \frac{-i\sqrt{95}}{2}$$

Figure 45. Examples of answers to control class students who get a score of 3 for question number 7

In Figures 44 and 45 it can be seen that students have been able to check the equation of tangents to a circle if a circle crosses the Y axis at a point without making conclusions at the end of the answer. So that students get a score of 3.

Handwritten mathematical work for Figure 46:

Diketahui:

$$x^2 + y^2 - 2x + 13y + 40 = 0$$

Ditanya: y?

Jawab: $y = -5 (0, -5)$ 2

Figure 46. Examples of answers of experimental class students who get a score of 2 for question number 7

Handwritten student work for question 7. The work includes several equations and a point:

$$Diketahui: x^2 + y^2 - 2x + 13y + 40 = 0$$

$$Ditanyakan: Persamaan garis singgung yang sejajar dengan garis $y = 2x + 5$ melalui titik $(0, -5)$

$$Jawab: Persamaan garis singgung yang sejajar dengan garis $y = 2x + 5$ melalui titik $(0, -5)$ adalah $y = 2x + 5$$$$$

Figure 47. Examples of answers to control class students who get a score of 2 for question number 7

In Figures 46 and 47, it can be seen that students have tried to check the equation of tangents to the circle if a circle crosses the Y axis at a point but students have not finished working on the answer. Answers like this get a score of 2.

Handwritten student work for question 7. The work includes a circle equation and a line equation:

$$Diketahui: x^2 + y^2 - 2x + 13y + 40 = 0$$

$$y = -5$$

$$3y - 2x + 15 = 0$$

Ditanyakan: Persamaan garis singgung yang sejajar dengan garis $y = 2x + 5$ melalui titik $(0, -5)$

Figure 48. Examples of answers of experimental class students who get a score of 1 for question number 7

In Figure 48 it appears that students have not been able to develop the necessary or sufficient conditions of a concept because it only mentions what is known in the questions so that students are given a score of 1.

B. Discussion

1. Development of Students' Mathematical Concept Understanding

The development of students' understanding of mathematical concepts during the *Think Talk Write* learning model has increased. The development of students' understanding of mathematical concepts is seen from the percentage

the number of students who got the maximum score in each indicator during the six meetings. The percentage of students who obtained the maximum score increased in each indicator.

The increase in the percentage of students who get the maximum score in each indicator is due to the application of the *Think Talk Write* learning model in the learning process. The results of research that have been conducted by researchers found that there was an increase in student ability which was marked by an increase in the number of students who obtained the maximum score in the work of Student Worksheet. This is in line with the results of research from Fitriyana, Nur & Rani Asnurida (2018) where in learning activities with the Think Talk Write (TTW) strategy, it is evident that students' understanding of mathematical concepts has begun to increase which is marked by the ability of students to solve problems in LAS well. In this study, there was also an increase in the average result of the mathematical concept understanding test in the experimental class compared to the control class. This is in line with the results of research by Suherman et al. (2018) which also revealed that the Think Talk Write (TTW) learning model had an influence on students' mathematical communication skills, where the average score of the mathematics communication skills test in the experimental class was higher than the control class. That is, for each indicator of mathematics communication skills, the experimental class students are better than the control class.

This TTW learning model has 3 learning stages, namely:

a. Think

Think it means to think. Thinking is the development of ideas and concepts in a person that involves the work of the brain. In Kamus Besar Bahasa Indonesia (KBBI) thinking means using reason to consider and decide something, as well as considering it in memory.

The Think or thinking stage can be seen through the process of reading a mathematical text, thinking about possible answers, and then making notes on what has been read. When making or writing these notes, students are required to be able to differentiate and unify the ideas presented in the reading text to be translated into their own language.

At this stage students will read a number of problems given on the Student Worksheet , then after reading the students will write down things that are known and unknown about the problem (making individual notes). Furthermore, students are asked to solve existing problems individually. The thought process at this stage will be seen when students read the problem and then write back what is known and what is not known about a problem.

b. Talk

Talk interpreted as speaking or discussing. At this stage students communicate using their own words and language.

The *talk* stage allows students to speak skillfully. At this stage students will practice mathematical communication with group members orally. The problem to be discussed is a problem that students have previously thought at the *think* stage.

The process of talking is learned by students through their life as individuals who interact with the social environment. By discussing it can increase the activities of students in the class. Communicating in discussions creates a learning environment that encourages students to communicate between other students and can improve students' understanding because when students discuss, students construct various ideas to be put forward.

Sugandi, Asep Ikin (2011) stated that talk is important in mathematics because it is the main way to learn about mathematics, forming ideas, improving and assessing the quality of thinking. At the *talk* stage, students work with their groups using Student Worksheets. Student Worksheet contains practice questions that students must do in groups. The importance of *talk* in learning is that it can build mutual understanding and knowledge through interaction and conversation among individuals in groups. Finally, it can provide solutions to the problems faced which lead to an agreement in formulating learning objectives to be achieved.

c. Write

At this stage, students write down the results of the discussion they get from the first and second stage activities. According to Huda (2015: 219), writing consists of the basic concepts used, the relationship with the previous material, the settlement strategy, and the solutions obtained. Meanwhile, according to Shield and Swinson in Yamin (2012: 87) states that writing in mathematics helps to realize one of the learning objectives, namely students' understanding of the material they are learning. In addition, each and Wisniowska in Yamin (2012: 88) suggests that writing activities for teachers can monitor students' mistakes, misconceptions, and students' conceptions of the same idea. As stated by Cahyani and Hodijah in Zulkarnaini (2011: 145) that: Writing skills are the most complex skills because writing is not just copying words and sentences, but also developing and expressing thoughts in an organized writing. According to Ambarsari, Heny et al (2018: 119) The Think Talk Write (TTW) model is more effective, especially for students with high reading habits. Using the Think Talk Write model provides students with a good way to explore their writing skills. This involves students actively in several activities during teaching writing. 119) Think Talk Write (TTW) model is more effective, especially for students with high reading habits. Using the Think Talk Write model provides students with a good way to explore their writing skills. This involves students actively in several activities during teaching writing. 119) Think Talk Write (TTW) model is more effective, especially for students with high reading habits. Using the Think Talk Write model provides students with a good way to explore their writing skills. This involves students actively in several activities during teaching writing.

Through these learning stages, students will be guided to find their own concepts or principles. This does not mean that students discover new things because they have been found by others

previous. Students are not left alone to learn to discover, but educators facilitate students. By integrating the initial knowledge that students have previously had with the concepts being learned will make students able to build new knowledge so as to improve students' understanding of mathematical concepts. Quizzes I to VI that have been carried out by students have increased on average because the average score of students has decreased only in quiz IV to quiz V. The increase is due to the application of the *Think Talk Write* learning model which has learning stages that are can develop students' understanding of mathematical concepts students.

Quiz I and quiz II are tested to classify objects based on whether or not the requirements that form the concept are met. From the quiz that was tested, it was found that students were able to classify objects based on whether the requirements that formed the concept were met or not, as seen from the average score of the students' quiz.

In the third and fourth quizzes, the average score for the students' quiz was higher than the previous quiz. This can be seen from the number of completeness of students and the average student is superior to other quizzes. The indicators tested are indicators that identify the properties of the operation or concept. However, on the V quiz, the average and percentage of students' completeness decreased. The indicators tested were also the same in quiz III and IV, namely identifying the characteristics of the operation or concept. This is because of the participants

Confused students to answer indicator 3, namely identifying the properties of the operation or concept. However, if the students are categorized as capable, underprivileged and incapable, the students are at a disadvantage, meaning that in the quiz the students answered more than 2 or 1

In quiz VI, the students' scores increased compared to the previous quiz. Likewise, the percentage of students completeness also experienced a high increase. Learners on developing indicators need and / or sufficient conditions for a concept to have good abilities. This is because students at the *think* stage, *talk* stage and *write* stage students are more serious in working on the problems that exist in the Student Worksheet.

Based on the discussion above, overall it can be seen that the average score of students 'quizzes and the percentage of students' quiz score completeness has increased. However, on the indicators identifying the properties of the operation or the concept the learners had difficulty. This can be seen during the learning process, namely at the *think* stage, students are confused in working on Student Worksheet so that students need a long time to find concepts. Steps taken by educators in overcoming this problem are to direct students together to find a concept of the material being studied. Overall, the *Think Talk Write* learning model can improve students' conceptual understanding, this is in line with Sugandi's research results, *Talk Write* provides a meaningful role in the achievement of communication skills and mathematical reasoning.

2. Comparison of Students' Mathematical Concept Understanding in Experiment Class and Control Class

The understanding of students' mathematical concepts in this study was seen through tests, both in the experimental class and in the control class which contained seven indicators of understanding mathematical concepts, namely restating the concepts that have been learned, classifying objects based on the requirements of the concept, identifying the characteristics of the nature of operations or concepts, applying a concept logically, giving examples or examples of cons (not examples) of the concepts being studied, linking various concepts in mathematical concepts and outside mathematics, developing the necessary and / or sufficient conditions of a concept. This test aims to determine whether the understanding of the mathematical concepts of students who learn using the *Think Talk Write* learning model is better than understanding the mathematical concepts of students who learn using the direct learning model.

Based on the data description and the results of the analysis of the mathematical concept understanding test data, it was found that the average test score for the experimental class was higher than the average test score for the control class. If seen from the average score of each indicator of the ability to understand mathematical concepts, the average score of the experimental class students is higher than the average score of the control class students. In addition, based on the hypothesis test, it was also found that the test results of students' mathematical concept understanding in the experimental class were better than

the results of the control class students' mathematical concept understanding test results. This shows that the understanding of the mathematical concepts of students who learn using the *Think Talk Write* learning model is better than understanding the mathematical concepts of students who learn using the direct learning model.

The following describes the stages of the *Think Talk Write* learning model which can affect the indicators of understanding students' mathematical concepts.

1. *Think* stage

This stage is the stage where students read problems about understanding mathematical concepts in the Student Worksheet and make important notes from the results of reading individually (*think*) for further discussion.



Figure 49. Learners Do the Think Phase

In the learning process, students respond to stimuli provided by the educator well, this is because the educator explains in detail the model used and its stages as well as explaining the Student Worksheet that will be carried out by students.

Then the teacher gave stimulation in the form of questions in the Student Worksheet that help students get something at the think stage individually.

2. Stage *Talk*

At this stage, students interact and collaborate with their group of friends to discuss the contents of notes and solutions to problems given in the Student Worksheet (talk).



Figure 50. Students Doing the Talk Stage

In each meeting, students are guided to communicate the concepts obtained in a material through the help of Student Worksheet with a group of friends. After students are given *talk* that increases indicators identifying the properties of operations or concepts, and also students are asked to find concepts from material tangent to circle equations which at this stage can also improve concept understanding indicators, namely presenting concepts in various forms of mathematical representation, then students are asked to restate the concepts that have been obtained from the material tangent to circle equations which can improve indicators.

Restate the concepts that have been learned. So, in the *Think Talk Write* learning model at the *talk* stage students can develop an understanding of their mathematical concepts, especially on indicators of restating a concept, presenting concepts in various forms of mathematical representation, and identifying the properties of operations or concepts.

3. *Write* stage

Educators provide opportunities for students to write the results of mutual agreement on solving problems in understanding mathematical concepts (*write*).



Figure 51. Students Do the Write Phase

At the writing stage, it can improve students' conceptual understanding on indicators presenting concepts from various forms of mathematical representation.

Every meeting of students is enthusiastic about doing the exercises, and students have carried out all stages correctly.

3. The relationship between students' mathematical concept understanding and data analysis results

Based on each data analysis result obtained, it is proven that the *Think Talk Write* learning model improves students' understanding of mathematical concepts. It can be seen from the results of the development of quizzes about understanding the mathematical concepts of students in the experimental class which generally show an increase even though they experience fluctuation because each meeting the level of difficulty of the questions is different. The results of the mathematical concept understanding test conducted at the end of the study also showed that the experimental class obtained higher results than the control class. Thus it can be concluded that the *Think Talk Write* learning model affects students' understanding of mathematical concepts. This is because the *Think Talk Write* learning model encourages students to think, speak,

C. Research Constraints

During the research, there were several obstacles that the researcher faced in implementing the *Think Talk Write* learning model. The obstacle is that students are not accustomed to using Student Worksheet and from insufficient time to do learning. During the research, there were several lessons that were not timed properly. This is because the stages in the *Think Talk Write* learning model takes a lot of time, especially at the think stage.

At the *think* stage, students are required to find the concept personally from the given Student Worksheet. To minimize this obstacle, researchers try to use time effectively for each stage of the *Think Talk Write* learning model and emphasize more on limiting time to complete Student Worksheet.

Another obstacle exists in the learning process using the *Think Talk Write* learning model. At the beginning of learning, students feel confused because they are not familiar with this learning model. Students also have difficulty understanding how to fill Student Worksheet, plus students are not familiar with using Student Worksheet in learning. To minimize this obstacle, at the beginning of the research meeting, the educator gave directions about the activities to be carried out during the learning process. Researchers also provide instructions to students in working on the given Student Worksheet so that at the next meeting students begin to understand what they have to do in the learning process.

In addition, the obstacle faced at each meeting is that there are still students who are not serious in following the learning being carried out. To overcome this, the researcher reminded students that in each lesson quizzes would be carried out and the quiz scores would later be entered into the grade book so that students became more serious and focused on learning so that they could answer quiz questions later and so they didn't get low scores and also each group that can complete the Student Worksheet fast, correct and complete learners give points to each group and give prizes in the form of food for the first group of collectors.

CHAPTER V CLOSING

A. Conclusion

1. The understanding of the mathematical concepts of students who learn to use the *Think Talk Write* learning model is better than understanding the mathematical concepts of students who learn using the direct learning model in class XI SMA Adabiah 2 Padang. This can be seen from the final test of students and the results of hypothesis testing, namely $P\text{-value} = 0.003$.
2. The development of students' understanding of mathematical concepts in class XI SMA Adabiah 2 Padang during the implementation of the *Think Talk Write* learning model has increased which can be seen in the percentage of students who get the maximum score in quizzes, Student Worksheet, and exercises on each indicator of understanding mathematical concepts. This means that the *Think Talk Write* learning model can be said to have an effect on students' understanding of mathematical concepts.

B. Suggestion

Based on the above conclusions, the following points are suggested:

1. Researchers should consider the time allocation required when students are in groups so that learning objectives can be achieved because the *Think Talk Write* learning model requires a lot of time.
2. Teachers in the field of mathematics studies at SMA Adabiah 2 Padang can make the results of this study as an alternative in a variety of learning, one of which is the *Think Talk Write* learning model
3. For the next researcher to be able to continue on the material and other mathematical abilities, as well as pay attention to the constraints that researchers experience in order to get better research results than the researchers did.

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Appendix 1

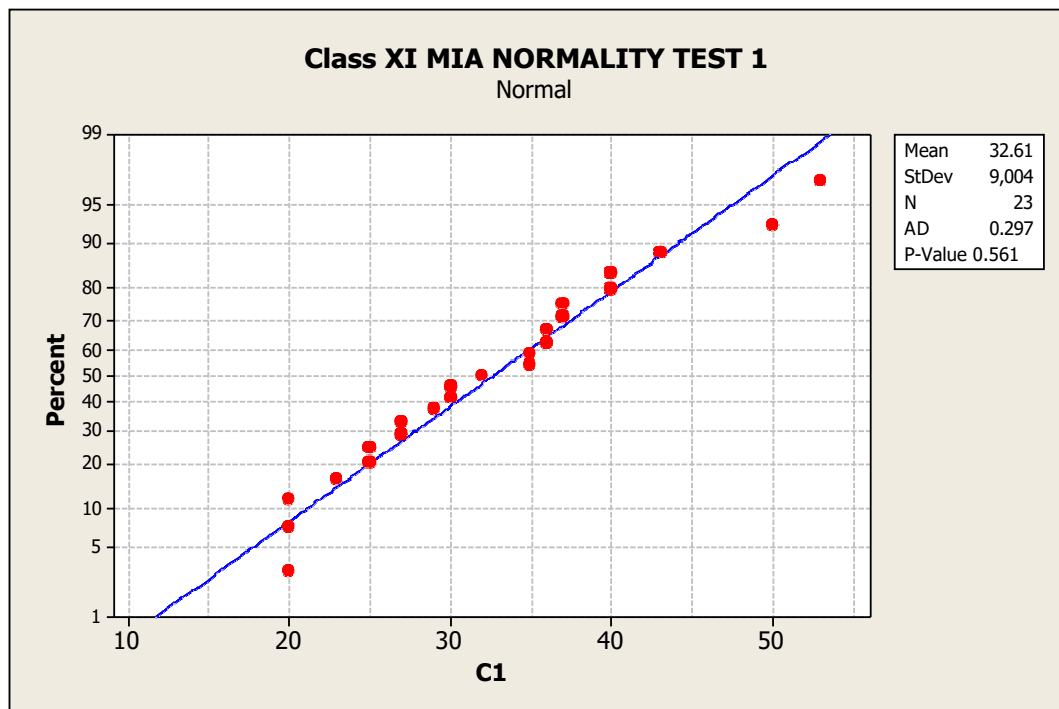
FINAL ASSESSMENT OF CLASS XI STUDENTS OF SMA ADABIAH 2 PADANG STUDY YEAR 2019/2020

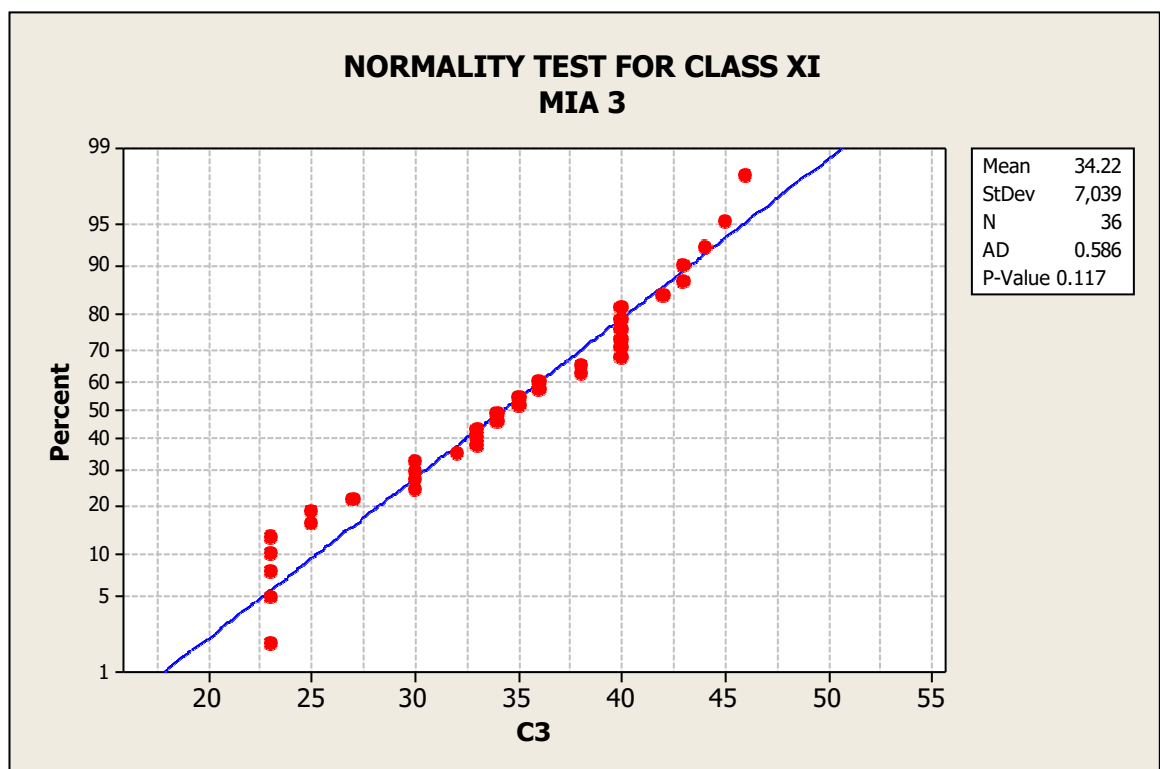
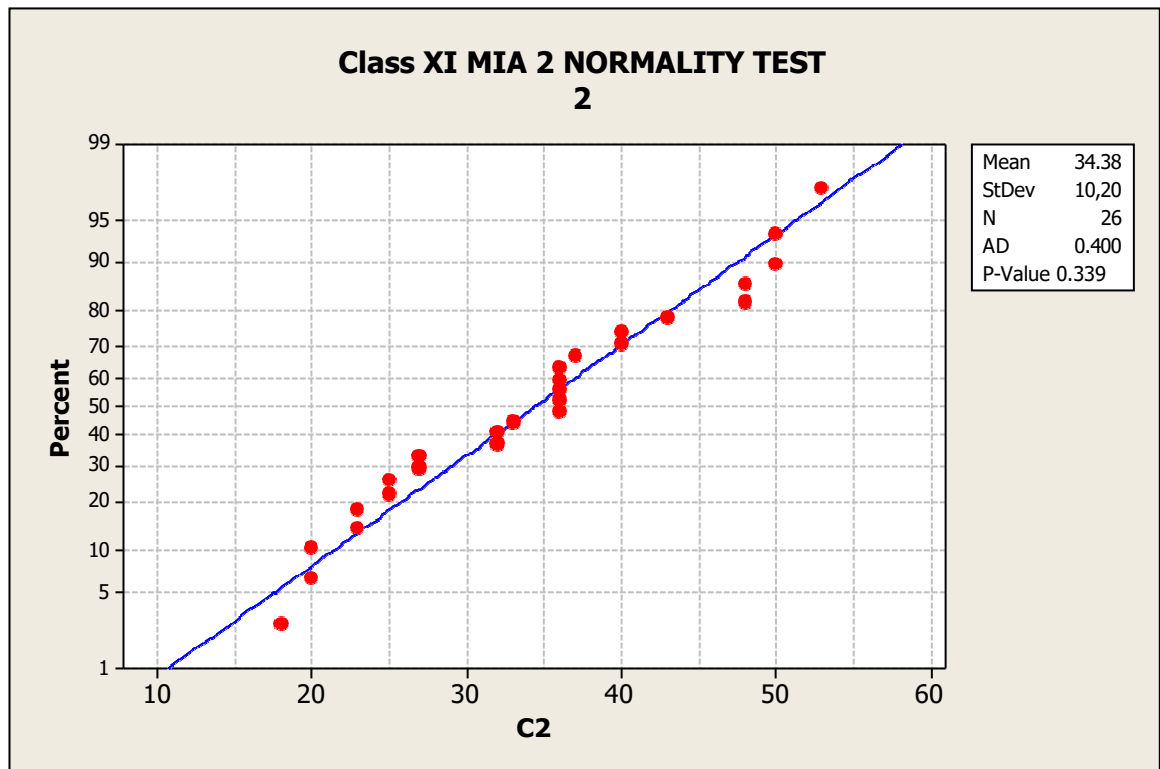
No.	Class			
	XI MIA 1	XI MIA 2	XI MIA 3	XI MIA 4
1	43	50	40	29
2	36	36	43	28
3	40	40	40	25
4	36	48	25	28
5	30	40	36	30
6	50	48	43	35
7	32	32	32	30
8	29	25	38	31
9	20	18	33	34
10	35	36	40	48
11	27	27	40	27
12	20	50	25	32
13	25	32	23	18
14	23	27	23	32
15	25	37	33	34
16	35	25	30	30
17	37	36	23	44
18	37	43	30	52
19	53	53	40	46
20	20	20	46	41
21	40	20	35	43
22	30	23	27	44
23	27	36	36	35
24		36	23	53
25		23	23	43
26		33	40	35
27			38	18
28			34	40
29			35	36
30			44	43
31			45	43
32			34	18
33			30	36

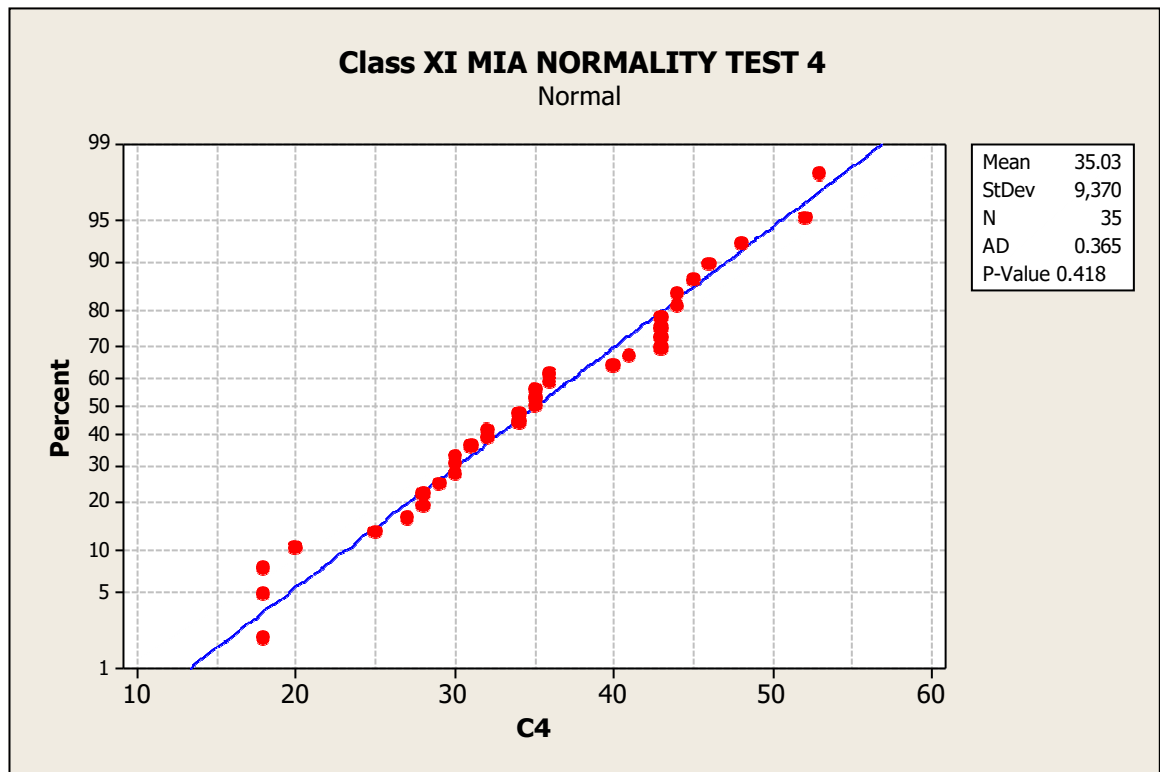
34			30	45
35			42	20
36			33	
Average	32.60	34.38	34.22	35.02

Appendix 2

POPULATION CLASS NORMALITY TEST

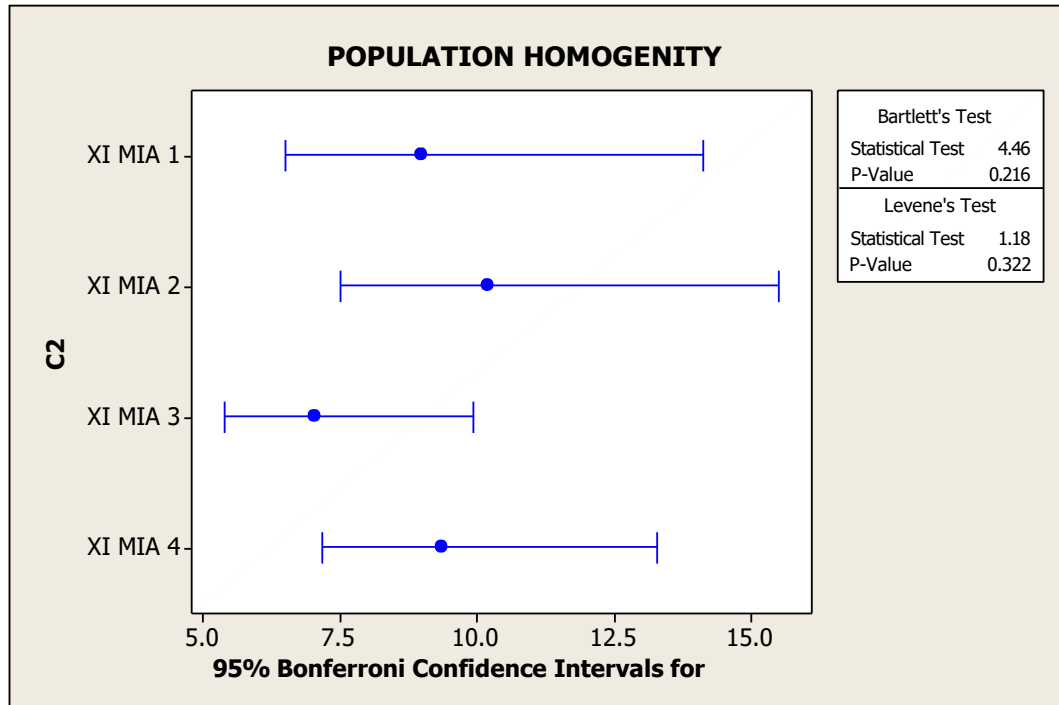






Appendix 3

POPULATION CLASS VARIANCE HOMOGENITY



Appendix 4

POPULATION CLASS AVERAGE EQUITY TEST

One-way ANOVA: XI MIA 1; XI MIA 2; XI MIA 3; XI MIA 4

Source	DF	SS	MS	F	P
C2	3	83.1	27.7	0.35	0.787
Error	116	9104.8	78.5		
Total	119	9188.0			

S = 8,859 R-Sq = 0.90% R-Sq (adj) = 0.00%

				Individual 5% CIs For Mean Based on Pooled StDev			
Level	N	Mean	StDev	-----+-----+-----+-----+-----			
XI MIA 1	23	32,609	9,004	(- *			
XI MIA 2	26	34,385	10,202		(* -)		
XI MIA 3	36	34,222	7,039		(*)		
XI MIA 4	35	35,029	9,370			(* -)	
				-----+-----+-----+-----+-----			
				32,90	33.60	34.30	35.00

Pooled StDev = 8,859

Appendix 5

LESSON PLAN VALIDATION SHEET

Education Unit : SMA Adabiah 2 Padang

Subjects : Mathematics

Class / Semester : XI / 2

Subject : Equations of Tangents to Circles

ASSESSMENT INSTRUCTIONS:

Put a check mark (✓) in the answer column below with the assessment criteria as follows:

Answer Column 1 = Strongly Disagree

Answer Column 2 = Disagree

Answer Column 3 = Agree

Answer column 4 = Strongly Agree

No	Rated aspect	Assessment				Note
		1	2	3	4	
1	General					
	a. Identity of subjects, KI, KD competency achievement indicators, learning objectives, teaching materials, time allocation, learning methods (introduction, core, cover), learning resources are clearly and completely stated				✓	
	b. Lesson Plans are prepared for each KD which can be implemented in one or more meetings			✓		
2	Lesson Plan Identity					
	c. Education units, classes, semesters, subjects, number of meetings have been written completely and clearly			✓		
3	Core Competencies (KI), and Basic Competencies (KD)					
	a. Core competencies (KI) and basic competencies (KD) are in accordance with content standards			✓		
	b. KD is related to KI				✓	

4	Indicator					
	a. Indicators developed are in accordance with basic competencies			✓		
	b. The indicators developed are in accordance with the characteristics of students, subjects, and educational units			✓		
	c. Indicators have been formulated using operational verbs that can be measured and observed, which include knowledge, skills and attitudes			✓		
	d. The operational verb (KKO) on the indicator of achievement is correct			✓		
5	Formulation of Learning Objectives					
	a. The learning objectives are in accordance with the basic competencies that will be achieved by students					
6	Learning materials			✓		
	a. The scope of learning material is in accordance with the competencies to be achieved			✓		
	b. The learning material has been presented systematically			✓		
	c. The learning material is in accordance with the time allocation			✓		
	d. The prerequisite material is in accordance with the main material to be studied			✓		
	e. The learning material presented contains facts, concepts, principles and relevant procedures			✓		
7	Suitability and diversity of learning resources					
	a. The learning resources used are varied			✓		
	b. Learning resources have supported the learning material			✓		
8	Time Allocation					
	a. Time allocation is in accordance with the need to achieve KD and the learning load			✓		
9	Learning Activities					
	a. Preliminary activities					

	Initial activities to provide motivation and focus the attention of students to actively participate in the learning process			✓		
	b. Core activities					
	It is a learning process to achieve learning objectives through a scientific approach (observing, questioning, gathering information, reasoning, concluding and communicating) and the PBL model, namely:			✓		
	c. Closing activities					
	1. Reflect on activities to end learning activities			✓		
	2. Make a summary or conclusion of the assessment			✓		
10	Assessment					
	A. The assessment techniques and instruments are complete			✓		

Validator recommendations for Lesson Plan.

.....

Padang, February 2020
Validator

(Dr. Yerizon, .M.Si)

Appendix 6

LEARNING IMPLEMENTATION PLAN (RPP)

EXPERIMENT CLASS

Education units	: Adabiah High School 2
Padang Subjects	: Math Class / semester
	: XI / II (Even)
Subject matter	: Tangent Equations Time Allocation
Circle	: 12 JP (6 Gatherings)

A. Core Competencies

1. Living and practicing the teachings of the religion they adhere to.
2. Demonstrate honest, disciplined, responsible, caring behavior (mutual cooperation, cooperation, tolerance, peace), courteous, responsive, and pro-active as part of the solution to various problems in interacting effectively with the social and natural environment and placing oneself as a reflection nation in the world association.
3. Understand, apply, analyze factual, conceptual, procedural knowledge based on curiosity about science, technology, arts, culture and humanities with insights into humanity, nationality, statehood and civilization related to the causes of phenomena and events, and apply procedural knowledge in the field of study specific talents and interests in solving problems.
4. Processing, reasoning, and presenting in the realm of concrete and abstract realms related to the development of what they learn in school independently, and being able to use methods according to scientific principles.

B. Basic Competencies and Competency Achievement Indicators

Basic competencies	
3.3 Analyze circles analytically	4.3 Resolves problems associated with circles
Indicators of Competence Achievement	
Meeting 1	
3.3.1 Determine line tangent circle	4.3.1 Solve problems related to tangents to circles
3.3.2 Analyze the equation for the tangent to the circle centered at point O (0,0) and the radius r if the slope is known	4.3.2 Solve problems related to the equation tangents to the circle centered at point O (0,0) and the radius r if the gradient is known contextually
Meeting-2	
3.3.3 Analyze the equation of the tangent to the circle centered at point P (a, b) and of radius r if the slope is known	4.3.3 Solve problems related to the equation of tangents to a circle centered at point P (a, b) and of radius r if the gradient is known contextually
Meeting-3	
3.3.4 Analyze the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$	4.3.4 Solve the problem related to the equation tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$ in a manner contextual
Meeting-4	
3.3.5 Analyze the equation for the tangent to the circle at point A (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$	4.3.5 Solve the problem related to the equation tangent to the circle at point A (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$ in a manner contextual
Meeting-5	
3.3.6 Analyze the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$	4.3.6 Solve the problem related to the equation tangent to the circle at point A (x_1, y_1) on the circle

	$x^2 + y^2 + Ax + By + C = 0$ in a manner contextual
Meeting-6	
3.3.7 Analyze the equation for the tangent to a circle at a point outside the circle	4.3.7 Contextually solve problems related to the equation tangent to a circle at a point outside the circle

C. Learning objectives

Meeting 1

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation of the tangent to the circle centered at point O (0,0) and the radius r if the slope is known
2. Analyze the equation for the tangent to the circle centered at point O (0,0) and the radius r if the slope is known

Meeting-2

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation for the tangent to the circle centered at point P (a, b) and the radius r if the slope is known
2. Analyze the equation of the tangent to the circle centered at point P (a, b) and the radius r if the slope is known

Meeting-3

Through learning activities using the *Think Talk Write* model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation tangent to the circle at point A (x_1, y_1)
on the circle $x^2 + y^2 = r^2$ appropriately
2. Analyze the equation for the tangent to the circle at point A (x_1, y_1)
on the circle $x^2 + y^2 = r^2$ appropriately

Meeting-4

Through learning activities using the *Think Talk Write* model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation tangent to the circle at point A (x_1, y_1)
on the circle $(x - a)^2 + (y - b)^2 = r^2$ appropriately
2. Analyze the equation for the tangent to the circle at point A (x_1, y_1)
on the circle $(x - a)^2 + (y - b)^2 = r^2$ appropriately

Meeting-5

Through learning activities using the *Think Talk Write* model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation tangent to the circle at point A (x_1, y_1)
on the circle $x^2 + y^2 + 2Ax + 2By + C = 0$ appropriately
2. Analyze the equation for the tangent to the circle at point A (x_1, y_1)
on the circle $x^2 + y^2 + 2Ax + 2By + C = 0$ appropriately

Meeting-6

Through learning activities using the *Think Talk Write* model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Analyze the equation for the tangent to a circle that is known to be a point outside the circle with precision
2. Solve the problem of the equation tangent to a circle which is known to be a point outside the circle exactly

B. Theory

Meeting 1 (3JP)

Fact :

Circle equation symbol: I

Variables: x, y

Coat of arms: r

Coat of arms of Gradeien: m

Concept:

Tangents to a circle are lines that intersect

circle at one point and perpendicular to the radius of the circle.

Principle:

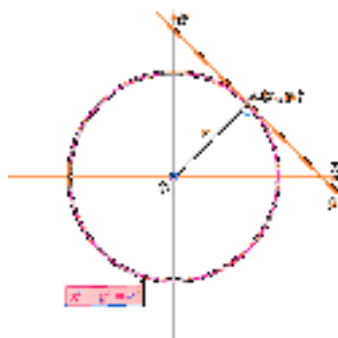
The equation for the tangent to the circle centered at point O (0,0) and the radius r if the slope is known $y = mx \pm r\sqrt{1+m^2}$.

Procedure:

Find the form of the equation tangent to the circle centered at O (0,0) and the radius r if the slope is known by solving the example below.

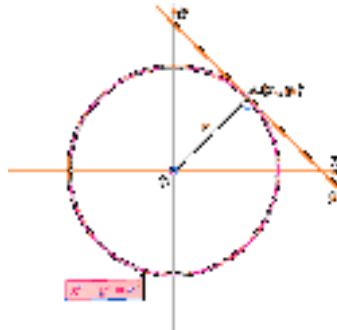
The definition of tangent to circle

Tangents to a circle are lines that intersect the circle at one point and are perpendicular to the radius of the circle. The point of intersection of the tangent and the circle is called the point of tangency.



In the image above, lines ℓ touching circle at point A (x_1, y_1) Line ℓ perpendicular to the radius of the circle OA. Point A (x_1, y_1) is called the point of contact.

Equations of Tangents to a Circle Centered at O (0,0) and Radius r If the Gradient is Known



There is a circle L that centered on $O(0,0)$ and radius with r the equation: $x^2 + y^2 = r^2$. And lines q that offending circle

L and graded m : $y = mx + k$

The equation tangents to the gradient m have shape $y = mx + k$ with m known and k can be obtained from substitution. The point where the lines fall with circle can be determined by substituting equation of the line to circular equation

Equation line substitution q : $y = mx + k$ to the circular equation

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = r^2$$

$$\Leftrightarrow x^2 + (mx + k)^2 = r^2$$

$$\Leftrightarrow x^2 + (mx + k)(mx + k) = r^2$$

$$\Leftrightarrow x^2 + m^2x^2 + mkx + mkx + k^2 - r^2 = 0$$

$$\Leftrightarrow (1 + m^2)x^2 + (2mk)x + (k^2 - r^2) = 0$$

$$\text{Where } a = 1 + m^2, b = 2mk \text{ and } c = k^2 - r^2$$

Because of the line q offend circle L , then the line q and circle L has only one common point. So it's a quadratic equation

$$(1 + m^2)x^2 + (2mk)x + (k^2 - r^2) \text{ has only one solution,}$$

then the value is 0

$$D = 0$$

$$b^2 - 4ac = 0$$

$$(2mk)^2 - 4(1 + m^2)(k^2 - r^2) = 0$$

$$4m^2k^2 - (4 + 4m^2)(k^2 - r^2) = 0$$

$$4m^2k^2 - (4k^2 - 4r^2 + 4m^2k^2 - 4m^2r^2) = 0$$

$$-4k^2 + 4r^2 + 4m^2r^2 = 0$$

$$-4(k^2 - r^2 - m^2r^2) = 0$$

$$k^2 - r^2 - m^2r^2 = 0$$

$$k^2 = r^2 + m^2r^2$$

$$k^2 = r^2(1 + m^2)$$

$$\sqrt{k^2} = \sqrt{r^2(1 + m^2)}$$

$$k = \pm r\sqrt{1 + m^2}$$

$$\Leftrightarrow k_1 = r\sqrt{1+m^2}$$

$$\Leftrightarrow k_2 = -r\sqrt{1+m^2}$$

So that the equation is tangent to \odot can be written Becomes

$$y = mx \pm r\sqrt{1+m^2}$$

Meeting-2 (2JP)

Facts:

Circle equation symbol: \odot

Variables: x, y

Coat of arms: r

Coat of arms of Gradeien: m

Concept:

Tangents to a circle are lines that intersect the circle at one point and are perpendicular to the radius of the circle.

Principle

The equation for the tangent to the circle centered at point P (a, b) and the radius r if the slope is known $y - b = m(x - a) \pm r\sqrt{1+m^2}$.

Procedure

Find the form of the equation tangent to the circle centered at P (a, b) and the radius r if the slope is known by solving the example below.

Equations of Tangents to a Circle Centered at P (a, b) and the Radius r If the Gradient is Known

The equation of the line with the slope m i.e. $y = mx + n$ (we will determine this value of n later). To find out the position of a line to the circle, substitute the line equation for the circle equation

$$L \equiv (x - a)^2 + (y - b)^2 = r^2$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - a)^2 + (mx + n - b)^2 = r^2$$

$$x^2 - 2ax + a^2 + m^2x^2 + 2m(n - b)x + (n - b)^2 - r^2 = 0$$

$$(m^2 + 1)x^2 + [2m(n - b) - 2a]x + (n - b)^2 + a^2 - r^2 = 0$$

$$\text{Where } a = m^2 + 1, b = [2m(n - b) - 2a] \text{ and } c = (n - b)^2 + a^2 - r^2$$

The condition for a line tangent to the circle is the discriminant value $\Delta = 0$

.Find the n value of the discriminant value of the line equation! (first find the discriminant of the equation you got in step 1).

$$\Delta = 0$$

$$b^2 - 4ac = 0$$

$$[2m(n - b) - 2a]^2 - 4(m^2 + 1)((n - b)^2 + a^2 - r^2) = 0$$

$$(b - am - n)^2 = r^2(1 + m^2)$$

$$b - am - n = \pm \sqrt{r^2(1 + m^2)}$$

$$b - am - n = \pm r\sqrt{1 + m^2}$$

$$n = b - am \pm r\sqrt{1 + m^2}$$

Substitute the value of n you got into the equation for the line $y = mx + n$,

so we get the equation tangent to the circle with the gradient m on the circle

$$(x - a)^2 + (y - b)^2 = r^2$$

$$y = mx + n$$

$$y = mx + b - am \pm r\sqrt{1 + m^2}$$

$$y - b = m(x - a) \pm r\sqrt{1 + m^2}$$

Meeting-3 (3JP)

Facts:

Circle equation symbol: \odot

Variables: x, y

Coat of arms: r

Coat of arms of Gradeien: m

Concept:

Tangents to a circle are lines that intersect the circle at one point

and are perpendicular to the radius of the circle.

Principle :

The equation tangents to the circle $x^2 + y^2 = r^2$ at some point on circle that is $x_1x + y_1y = r^2$.

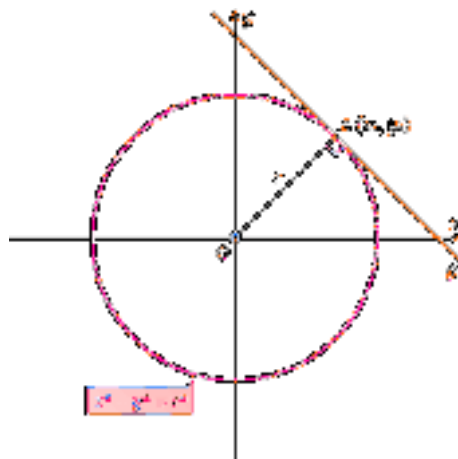
Procedure :

Find shape equation line tangent circle

$x^2 + y^2 = r^2$ at a point on the circle by completing example below.

Equation of Tangents to Circle at Point A (x_1, y_1) on the Circle

$$x^2 + y^2 = r^2$$



Look at the picture above!

Point A lies on the circle $x^2 + y^2 = r^2$. Line \overline{OA} has a gradient

$$m_1 = \frac{y_1}{x_1}.$$

Suppose a tangent ℓ have gradient m_2 . Because of the line

is a tangent $\overline{OA} \perp \ell$ so it applies $m_1 \times m_2 = -1$.

$$\text{Therefore } m_2 = \frac{-1}{m_1} = \frac{-x_1}{y_1}$$

Hence, the equation of the line ℓ that has a gradient $m_2 = \frac{-x_1}{y_1}$ and

through the point $A(x_1, y_1)$ is

$$y - y_1 = m_2(x - x_1)$$

$$\Leftrightarrow y - y_1 = \frac{-x_1}{y_1}(x - x_1)$$

$$\Leftrightarrow y_1 y - y_1^2 = -x_1 x + x_1^2$$

$$\Leftrightarrow x_1 x + y_1 y = x_1^2 + y_1^2$$

Because of the point of contact $A(x_1, y_1)$ lies on the circle $x^2 + y^2 = r^2$

then it applies $x_1^2 + y_1^2 = r^2$. As a result, the equation of the line is

$$x_1 x + y_1 y = r^2.$$

Meeting-4 (2JP)

Facts:

Circle equation symbol: \odot

Variables: x, y

Coat of arms: r

Coat of arms of Gradeien: m

Concept:

Tangents to a circle are lines that intersect the circle at one point and are perpendicular to the radius of the circle.

Principle :

The equation tangents to the circle $(x - a)^2 + (y - b)^2 = r^2$ at a point on the circle ie $(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$.

Procedure:

Determine the equation for the tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$ at a point on the circle by completing the example below.

Equation of Tangents to Circle at Point A (x_1, y_1) on the Circle

$$(x - a)^2 + (y - b)^2 = r^2$$



Line gradient \overline{AP} is $m_1 = \frac{y_1 - b}{x_1 - a}$

Suppose a line gradient q is m_2 . Because of the \overline{AP} and q , applies

$$m_1 \times m_2 = -1 \Leftrightarrow m_2 = \frac{-1}{m_1} = -\frac{x_1 - a}{y_1 - b}$$

Hence, the equation of the line q that has a gradient $m_2 = -\frac{x_1 - a}{y_1 - b}$ and

through the point $P(x_1, y_1)$ is

$$y - y_1 = m_2(x - x_1)$$

$$\Leftrightarrow y - y_1 = -\frac{x_1 - a}{y_1 - b}(x - x_1)$$

$$\Leftrightarrow (y - y_1)(y_1 - b) = -(x_1 - a)(x - x_1)$$

$$\Leftrightarrow y_1 y - y_1^2 - by + by_1 = -(x_1 x + x_1^2 + ax - ax_1)$$

$$\Leftrightarrow x_1 x - ax + ax_1 + y_1 y - by + by_1 = x_1^2 + y_1^2 \text{ (Equation 1)}$$

Because point $P(x_1, y_1)$ lies on the circle, then substitute a point $P(x_1, y_1)$ to circle:

$$(x - a)^2 + (y - b)^2 = r^2, \text{ obtained:}$$

$$\Leftrightarrow (x_1 - a)^2 + (y_1 - b)^2 = r^2$$

$$\Leftrightarrow x^2 - 2ax_1 + a^2 + y_1^2 - 2by_1 + b^2 = r^2$$

$$\Leftrightarrow x_1^2 + y_1^2 = r^2 + 2ax_1 - a^2 + 2by_1 - b^2$$

Substitute the form above into equation 1:

$$x_1x - ax + ax_1 + y_1y - by + by_1 = x_1^2 + y_1^2$$

$$\Leftrightarrow x_1x - ax + ax_1 + y_1y - by + by_1 = r^2 + 2ax_1 - a^2 + 2by_1 - b^2$$

$$\Leftrightarrow (x_1x - ax - ax_1 + a^2) + (y_1y - by - by_1 + b^2) = r^2$$

$$\Leftrightarrow (x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$$

Meeting-5 (2JP)

Facts:

Circle equation symbol: l

Variables: x, y

Coat of arms: r

Coat of arms of Gradeien: m

Concept:

Tangents to a circle are lines that intersect the circle at one point and are perpendicular to the radius of the circle.

Principles:

The equation tangents to the circle $x^2 + y^2 + Ax + By + C = 0$ in a

the point on the circle ie $x_1x + y_1y + \frac{A}{2}(x + x_1) + \frac{B}{2}(y + y_1) + C = 0$.

Procedure:

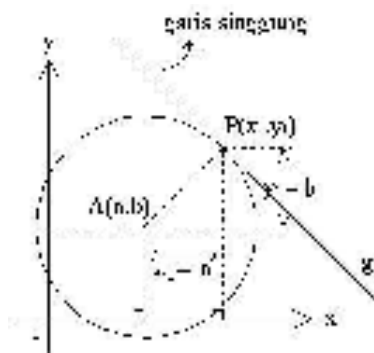
Determine equation line tangent circle

$x^2 + y^2 + Ax + By + C = 0$ in something point on circle with

complete the example below.

Equation of Tangents to Circle at Point A (x_1, y_1) on the Circle

$$x^2 + y^2 + Ax + By + C = 0$$



To determine the equation for the tangent to the circle at point a (x_1, y_1) on the circle

$x^2 + y^2 + Ax + By + C = 0$ simply describe the tangent

$$(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$$

$$\Leftrightarrow (x_1x - ax - ax_1 + a^2) + (y_1y - by - by_1 + b^2) = r^2$$

$$\Leftrightarrow x_1x + y_1y - a(x_1 + x) - b(y_1 + y) + a^2 + b^2 - r^2 = 0$$

Substitute it $a = -\frac{A}{2}$, $b = -\frac{B}{2}$, and $C = a^2 + b^2 - r^2$

$$\Leftrightarrow x_1x + y_1y - \left(-\frac{A}{2}\right)(x_1 + x) - \left(-\frac{B}{2}\right)(y_1 + y) + C = 0$$

$$\Leftrightarrow x_1x + y_1y + \frac{A}{2}(x_1 + x) + \frac{B}{2}(y_1 + y) + C = 0$$

Meeting-6 (2JP)

Facts:

Circle equation symbol: I

Variables: x, y

Coat of arms: r

Coat of arms of Gradeien: m

Concept:

Tangents to a circle are lines that intersect the circle at one point and are perpendicular to the radius of the circle.

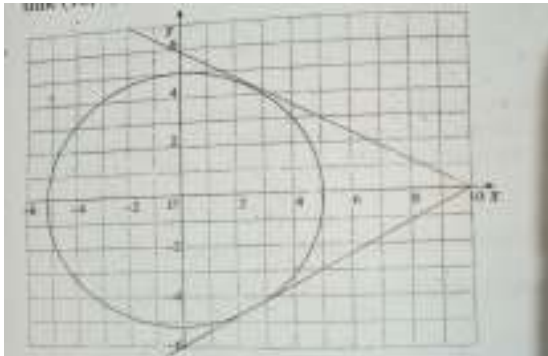
Principles:

The equation for the tangent to the circle at a point outside the circle

Procedure:

Determine the equation for the tangent to the circle at a point outside the circle by completing the example below.

Equation of Tangents to a Circle at a Point Outside the Circle



Determine the equation for the tangent to the circle $x^2 + y^2 = 25$ drawn from a point $(10,0)$ outside the circle.

Settlement

Suppose the gradient is tangent m and through point $(10, 0)$ outside the circle.

$$y = mx - mx_1 + y_1 \Leftrightarrow y = mx - m(10) + 0 \Leftrightarrow y = mx - 10m.$$

Step 1:

Substitute it $y = mx - 10m$ to the circular equation $x^2 + y^2 = 25$

$$x^2 + (mx - 10m)^2 = 25$$

$$\Leftrightarrow x^2 + (m^2x^2 - 20m^2x + 100m^2) - 25 = 0$$

$$\Leftrightarrow (1 + m^2)x^2 - 20m^2x + (100m^2 - 25) = 0$$

Step 2:

Discriminant value $D = b^2 - 4ac$

$$= (-20m^2)^2 - 4(1 + m^2)(100m^2 - 25)$$

$$= 400m^4 - 400m^2 + 100 - 400m^4 + 100m^2$$

$$= -300m^2 + 100$$

$$D = 0 \Leftrightarrow -300m^2 + 100 = 0$$

$$\Leftrightarrow 300m^2 = 100$$

$$\Leftrightarrow m^2 = \frac{1}{3}$$

$$\Leftrightarrow m = \pm \frac{1}{\sqrt{3}}$$

$$\Leftrightarrow m = \pm \frac{1}{3}\sqrt{3} \Leftrightarrow m_1 = \frac{1}{3}\sqrt{3} \text{ and } m_2 = -\frac{1}{3}\sqrt{3}$$

Step 3:

Substitute it m_1 and m_2 to $y = mx - 10m$

So, the equation of the tangent in question is

$$y = \frac{1}{3}\sqrt{3}x - 10\left(\frac{1}{3}\sqrt{3}\right) \Leftrightarrow 3y - \sqrt{3}x + 10\sqrt{3} = 0 \text{ and}$$

$$y = -\frac{1}{3}\sqrt{3}x - 10\left(-\frac{1}{3}\sqrt{3}\right) \Leftrightarrow 3y + \sqrt{3}x - 10\sqrt{3} = 0$$

C. Approaches, Models and Methods of

Learning Approaches : Scientific

Model : *Think Talk Write*

Method : Discussion, Question and Answer, Presentation

D. Media and Tools

Student Participants Worksheet (LKPD)

E. Learning Resources

1. Ministry of Education and Culture, Revised Edition. 2014.

Mathematics Student Book Jakarta: Ministry of Education and Culture

2. Ministry Education and Culture. 2017. Book

Students Mathematics Jakarta: Ministry of Education and Culture.

3. Ministry Education and Culture. 2017. Book

Teacher Mathematics Jakarta: Ministry of Education and Culture

F. Learning Activities

First meeting (2JP)

Description of Learning Activities	Time Allocation	TTW stage
<p>Preliminary activities</p> <ol style="list-style-type: none"> 1. Educators open the meeting by saying greetings 2. Educators together with students read prayers, Asma'ul Husna, and read the Koran (if during the first lesson) and check the attendance of students. <p>Phase 1: Conveying Objectives and Preparing Students</p> <ol style="list-style-type: none"> 3. Educators prepare students to learn. 4. The educator explains the learning procedure using the TTW (Think Talk Write) cooperative model: <ol style="list-style-type: none"> a. <i>Students will discuss in groups that will be divided by the teacher.</i> b. <i>One of the groups will be selected to present the results of the discussion to the class.</i> 	10 minutes	

<p>5. Educators inform the learning objectives and learning outcomes that are expected to be achieved.</p> <p>6. Apperception: Educators conduct questions and answers with students about material related to learning to be studied <i>Do you still remember the position of the line on the circle?</i></p> <p>7. Motivation: Educators conduct questions and answers with students about the benefits of studying the material being studied <i>Syria and Turkey have returned to turmoil after Turkish soldiers were killed in a shootout that took place in Idlib, northwestern Syria. Where the Turkish army intends to launch an attack on Syria, but Syria has a radar capable of detecting all movements 5 km in all directions and is at the center of coordinates. If you want to pass through the area, you can determine the direction taken so that it is not detected by the radar by studying the equation of the line tangent to the circle centered at point O (0,0) and of radius r if the gradient is known</i></p>		
<p>Core activities Phase 2: Presenting Information</p> <p>8. Educators provide information and problems related to the definition of tangents to a circle and the equation of a tangent to a circle centered at point O (0,0) and radius r if the gradient is known to students <i>Tangents to a circle are lines that intersect the circle at one point and are perpendicular to the radius of the circle. There is a circle L centered at O (0,0) and radius r with equation $x^2 + y^2 = r^2$. And the line g offends circle L and gradient m, then we can determine the equation of tangents if we know the equation of the circle $x^2 + y^2 = 16$ which is centered on point O (0,0) and has a gradient $2\sqrt{2}$.</i></p> <p>It is expected that students will be able to solve these problems using a formula $y = mx \pm r\sqrt{1 + m^2}$.</p>	<p>65 minutes</p>	

<p>17. Educators ask students to do activity 1 contained in the Student Worksheet in groups</p> <p>Communicating</p> <p>18. Educators randomly select groups that will present the results of the discussion in front of the class</p> <p>19. Educators ask students to write down the conclusions obtained at the Think and Talk stage in the language they understand.</p> <p>Phase 5: Evaluating</p> <p>20. Educators provide confirmation and reinforcement of the results of discussion and presentation of students and provide additional material explanation if something is missing</p>		
<p>Closing Activities</p> <p>Phase 6: Providing Recognition or Award</p> <p>21. Educators give awards to the best group at this meeting</p> <p>22. The educator together with the students concludes the lessons for this meeting</p> <p><i>The equation for the tangent to the circle centered at point $O(0,0)$ and the radius r if the slope is known $y = mx \pm r\sqrt{1+m^2}$.</i></p> <p>23. Educators give quizzes to students regarding the material they have just learned</p> <p>24. Educators give homework to students regarding the material at this meeting on the mathematics perspective student book page 81 numbers 1 and 3</p> <p>25. The educator asks students to read the next material about the equation tangent to a circle centered at point $P(a, b)$ and the radius of r if the gradient is known in the mathematics perspective student book</p> <p>26. The educator closes the lesson by saying hamdalah</p>	<p>15 minutes</p>	

Second meeting (2JP)

Description of Learning Activities	Time Allocation	TTW stage
<p>Preliminary activities</p> <ol style="list-style-type: none"> 1. Educators open the meeting by saying greetings 2. Educators together with students read prayers, Asma'ul Husna, and read the Koran (if during the first lesson) and check the attendance of students. <p>Phase 1: Conveying Objectives and Preparing Students</p> <ol style="list-style-type: none"> 3. Educators prepare students to learn. 4. The educator explains the learning procedure using the TTW (Think Talk Write) cooperative model: <ol style="list-style-type: none"> a. <i>Students will discuss in groups that will be divided by the teacher.</i> b. <i>One of the groups will be selected to present the results of the discussion to the class.</i> 5. Educators inform the learning objectives and learning outcomes that are expected to be achieved. 6. Apperception: Educators conduct questions and answers with students about material related to learning to be studied <i>Do you still remember how to determine the equation for the tangent to a circle centered at point $O(0,0)$ and the radius r if you know the gradient</i> 7. Motivation: Educators conduct questions and answers with students about the benefits of studying the material being studied <i>There have been severe forest fires in Australia in recent weeks. The local fire department has built a security base 1 km to the timu and 3 km to the north, the base is equipped with a radar that can detect forest fires with a radius of 3 km. If the base wants to extinguish the fire with a seaplane, it can determine the path to be taken in order to extinguish the fire right at the border area by studying the equation of the line</i> 	<p>10 minutes</p>	

<p><i>tangent to the circle centered at point P (a, b) and the radius r if the gradient is known</i></p>		
<p>Core activities</p> <p>Phase 2: Presenting Information</p> <p>8. Educators provide information and problems related to the equation tangent to a circle centered at point P (a, b) and the radius r if the gradient is known to students</p> <p><i>Determine the equation for the tangent to the circle $x^2 + y^2 - 6x + 10y - 46 = 0$ which is graded 2!</i></p> <p>It is expected that students will be able to solve these problems using a formula</p> $y - b = m(x - a) \pm r\sqrt{1 + m^2}.$ <p>Observe</p> <p>9. Students are asked to pay attention to the problems given by the educator</p> <p><i>Determine the equation for the tangent to the circle $x^2 + y^2 - 6x + 10y - 46 = 0$ which is graded 2!</i></p> <p>Ask</p> <p>10. Educators ask students to make questions about the problems that educators provide.</p> <p><i>How to determine the equation for the tangent to a circle centered at point P (a, b) and the radius r if the slope is known</i></p> <p>11. Educators say that students will be able to answer this question</p> <p>Phase 3: Organizing students into learning teams</p> <p>12. Educators form groups of 4-5 people and condition the students to sit in the group.</p> <p>13. Educators distribute Student Worksheet to all students</p> <p>14. Educators explain the steps that students will take in working on Student Worksheet</p> <p>Phase 4: Helping Teamwork and Learning to Experiment</p> <p>15. Educators ask students to observe activities</p>	<p>65 minutes</p>	<p>Step Think</p> <p>Step Talk</p>

<p>1 to determine the equation for the tangent to the circle centered at P (a, b) and the radius of r if the gradient is known and solve the problems given in the Student Worksheet individually and mark things that are not understood</p> <p>Reasoning</p> <p>16. Educators provide opportunities for students to communicate or discuss in their groups the results of their observations in activity 1 contained in Student Worksheet</p> <p>17. Educators ask students to work on questions in activity 1 in the Student Worksheet in groups</p> <p>Communicating</p> <p>18. Educators randomly select groups that will present the results of the discussion in front of the class</p> <p>19. Educators ask students to write down the conclusions obtained at the Think and Talk stage in the language they understand.</p> <p>Phase 5: Evaluating</p> <p>20. Educators provide confirmation and reinforcement of the results of discussion and presentation of students and provide additional material explanation if something is missing</p>		<p>Step Write</p>
<p>Closing Activities</p> <p>Phase 6: Providing Recognition or Award</p> <p>21. Educators give awards to the best group at this meeting</p> <p>22. The educator together with the students concludes the lessons for this meeting</p> <p><i>The equation for the tangent to the circle centered at point P (a, b) and the radius r if the slope is known $y - b = m(x - a) \pm r\sqrt{1 + m^2}$.</i></p> <p>23. Educators give quizzes to students regarding the material they have just learned</p> <p>24. The educator gives homework to the participants</p>	<p>15 minutes</p>	

<p>students related to the material at this meeting in the mathematics perspective student book page 81 numbers 2 and 4</p> <p>25. The educator asks students to read the next material about the equation tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$ on the mathematics perspective student book</p> <p>26. The educator closes the lesson by saying hamdalah</p>		
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Third meeting (2JP)

Description of Learning Activities	Allocation Time	Step TTW
<p>Preliminary activities</p> <ol style="list-style-type: none"> Educators open the meeting by saying greetings Educators together with students read prayers, Asma'ul Husna, and read the Koran (if during the first lesson) and check the attendance of students. <p>Phase 1: Conveying Objectives and Preparing Students</p> <ol style="list-style-type: none"> Educators prepare students to learn. The educator explains the learning procedure using the TTW (Think Talk Write) cooperative model: <ol style="list-style-type: none"> <i>Students will discuss in groups that will be divided by the teacher.</i> <i>One of the groups will be selected to present the results of the discussion to the class.</i> Educators inform learning objectives and 	10 minutes	

<p>expected learning outcomes will be achieved.</p> <p>6. Apperception: Educators conduct questions and answers with students about material related to learning to be studied <i>Do you still remember the equation for the tangent to a circle centered at point $P(a, b)$ and the radius r if you know the slope?</i></p> <p>7. Motivation: Educators conduct questions and answers with students about the benefits of studying the material being studied <i>The war between Turkey and Syria has never ended. Now, Turkey intends to launch an attack on Syria due to the death of Turkish soldiers in a shootout that took place in Idlib, northwestern Syria. Syrian army forces build city base with coordinates $(1,2)$. The base is equipped with a radar that can detect enemies with a radius of 25 km. The plane path that must be taken by the Turkish army troops in order to bomb the Syrian base at coordinate $(8,26)$ and not detected by radar can be determined by studying the equation of the tangent to the circle at point $A(x_1, y_1)$ on the circle $x^2 + y^2 = r^2$</i></p>		
<p>Core activities Phase 2: Presenting Information</p> <p>8. Educators provide information and problems related to the tangent to circle equations $x^2 + y^2 = r^2$ somewhere on the circle to learners <i>Find the equation for the tangent to the circle passing through point $(2,0)$ to center $P(0,0)$ and of radius 3!</i> It is expected that students will be able to solve these problems using a formula $x_1x + y_1y = r^2$</p> <p>Observe</p> <p>9. Students are asked to pay attention to the problems given by the educator <i>Find the equation for the circular lead line passing through the point $(2,0)$ with center $P(0,0)$ and of radius 3!</i></p> <p>Ask</p>	<p>65 minutes</p>	

<p><i>Talk in a language they understand.</i></p> <p>Phase 5: Evaluating</p> <p>20. Educators provide confirmation and reinforcement of the results of discussion and presentation of students and provide additional material explanation if anything is missing</p>		
<p>Closing Activities</p> <p>Phase 6: Providing Recognition or Award</p> <p>21. Educators give awards to the best group at this meeting</p> <p>22. The educator together with the students concludes the lessons for this meeting</p> <p><i>The equation tangents to the circle $x^2 + y^2 = r^2$ at a point on the circle ie $x_1x + y_1y = r^2$.</i></p> <p>23. Educators give quizzes to students regarding the material they have just learned</p> <p>24. Educators give homework to students related to the material at this meeting in the mathematics perspective student book page 78 numbers 1a and 1b</p> <p>25. The educator asks students to read the next material about the equation tangent to the circle at point A (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$ on the mathematics perspective student book</p> <p>26. The educator closes the lesson by saying hamdalah</p>	15 minutes	

The fourth meeting (2JP)

Description of Learning Activities	Time Allocation	TTW stage
<p>Preliminary activities</p> <p>1. Educators open the meeting by saying greetings</p> <p>2. Educators together with students read prayers, Asma'ul Husna, and read the Koran (if during the first lesson) and check</p>	10 minutes	

<p>the presence of students.</p> <p>Phase 1: Conveying Objectives and Preparing Students</p> <ol style="list-style-type: none"> Educators prepare students to learn. The educator explains the learning procedure using the TTW (Think Talk Write) cooperative model: <ol style="list-style-type: none"> <i>Students will discuss in groups that will be divided by the teacher.</i> <i>One of the groups will be selected to present the results of the discussion to the class.</i> Educators inform the learning objectives and learning outcomes that are expected to be achieved. Apperception: Educators conduct questions and answers with students about material related to learning to be studied <i>Do you still remember the equation tangent to the circle $x^2 + y^2 = r^2$ somewhere on the circle</i> Motivation: Educators conduct questions and answers with students about the benefits of studying the material being studied <i>There have been severe forest fires in Australia in recent weeks. The local fire department built a security base with coordinates (1, -2), the base is equipped with a radar that can detect forest fires with a radius of 15 km. If the base wants to extinguish the fire with a seaplane at point (10,10) then it can be determined the path that must be taken in order to extinguish the fire by studying the equation of the tangent to the circle at point A (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$</i> 		
<p>Core activities</p> <p>Phase 2: Presenting Information</p> <ol style="list-style-type: none"> Educators provide information and problems related to the tangent to circle equations $(x - a)^2 + (y - b)^2 = r^2$ at a point on the circle to the learners 	<p>65 minutes</p>	

<p><i>Find the equation for the tangent to the circle through the point (2,4) with the equation of the circle is $(x - 1)^2 + (y - 2)^2 = 5$!</i></p> <p>It is expected that students will be able to solve these problems using a formula $(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$</p> <p>Observe</p> <p>9. Students are asked to pay attention to the problems given by the educator</p> <p><i>Find the equation for the tangent to the circle through the point (2,4) with the equation of the circle is $(x - 1)^2 + (y - 2)^2 = 5$!</i></p> <p>Ask</p> <p>10. Educators ask students to make questions about the problems that educators provide.</p> <p><i>How to determine the equation for the tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$ in a point on the circle</i></p> <p>11. Educators say that students will be able to answer this question</p> <p>Phase 3: Organizing students into learning teams</p> <p>12. Educators form groups of 4-5 people and condition the students to sit in the group.</p> <p>13. Educators distribute Student Worksheet to all students</p> <p>14. Educators explain the steps that students will take in working on Student Worksheet</p> <p>Phase 4: Helping Teamwork and Learning to Experiment</p> <p>15. The educator asks students to observe activity 1 to determine the formula for the equation tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$ at some point on the circle and solve the problems given in the Student Worksheet individually and mark things that are not understood</p>		<p>Step Think</p> <p>Step Talk</p> <p>Step Write</p>
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<p>Reasoning</p> <p>16. Educators provide opportunities for students to communicate or discuss in their groups the results of their observations in activity 1 contained in Student Worksheet</p> <p>17. Educators ask students to work on questions in activity 1 in the Student Worksheet in groups</p> <p>Communicating</p> <p>18. Educators randomly select groups that will present the results of the discussion in front of the class</p> <p>19. Educators ask students to write down the conclusions obtained at the Think and Talk stage in the language they understand.</p> <p>Phase 5: Evaluating</p> <p>20. Educators provide confirmation and reinforcement of the results of discussion and presentation of students and provide additional material explanation if something is missing</p>		
<p>Closing Activities</p> <p>Phase 6: Providing Recognition or Award</p> <p>21. Educators give awards to the best group at this meeting</p> <p>22. The educator together with the students concludes the lessons for this meeting</p> <p><i>The equation tangents to the circle</i> $(x - a)^2 + (y - b)^2 = r^2$ at a point on the circle ie $(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$.</p> <p>23. Educators give quizzes to students regarding the material they have just learned</p> <p>24. Educators give homework to students regarding the material at this meeting on page 81 mathematics perspective student books numbers 1a, 1b and 2</p> <p>25. Educators ask students to read the next material regarding tangent equations</p>	<p>15 minutes</p>	

<p>circle at point A (x_1, y_1) on the circle</p> <p>$x^2 + y^2 + Ax + By + C = 0$ on the mathematics perspective student book</p> <p>26. Educators close the lesson by saying hamdalah</p>		
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The fifth meeting (2JP)

Description of Learning Activities	Time Allocation	TTW stage
<p>Preliminary activities</p> <ol style="list-style-type: none"> Educators open the meeting by saying greetings Educators together with students read prayers, Asma'ul Husna, and read the Koran (if during the first lesson) and check the attendance of students. <p>Phase 1: Conveying Objectives and Preparing Students</p> <ol style="list-style-type: none"> Educators prepare students to learn. The educator explains the learning procedure using the TTW (Think Talk Write) cooperative model: <ol style="list-style-type: none"> <i>Students will discuss in groups that will be divided by the teacher.</i> <i>One of the groups will be selected to present the results of the discussion to the class.</i> Educators inform the learning objectives and learning outcomes that are expected to be achieved. Apperception: Educators conduct questions and answers with students about material related to learning to be studied <i>Do you still remember the equation tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$ at some point on a circle?</i> Motivation: Educators conduct questions and answers with students about the benefits of studying the material being studied <i>In recent weeks, earthquakes have occurred frequently, especially in the city of Padang. If the point</i> 	10 minutes	

<p><i>the center of an earthquake on the map coordinates (3,7) and the earthquake has a radius of 36 km, so it can be determined whether a place outside the city of Padang that has coordinates (33,25) also feels the earthquake or not by studying the equation of the tangent to the circle in point A (x₁,y₁) on the circle</i></p> $x^2 + y^2 + Ax + By + C = 0$		
<p>Core activities</p> <p>Phase 2: Presenting Information</p> <p>8. Educators provide information and problems related to the tangent to circle equations $x^2 + y^2 + Ax + By + C = 0$ at a point on the circle to the learners</p> <p><i>To determine the equation for the tangent to the circle at point a (x₁,y₁) on the circle</i></p> <p>$x^2 + y^2 + Ax + By + C = 0$ simply describe the tangent</p> $(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$ <p><i>Determine the equation for the tangent to the circle $x^2 + y^2 - 6x + 8y - 120 = 0$ through the dots (15, -5)!</i></p> <p>It is expected that students will be able to solve these problems using a formula</p> $x_1x + y_1y + \frac{A}{2}(x - x_1) + \frac{B}{2}(y - y_1) + C = 0$ <p>Observe</p> <p>9. Students are asked to pay attention to the problems given by the educator</p> <p><i>Determine the equation for the tangent to the circle $x^2 + y^2 - 6x + 8y - 120 = 0$ through the dots (15, -5)!</i></p> <p>Ask</p> <p>10. Educators ask students to make questions about the problems that educators provide.</p> <p><i>How to determine the equation for the tangent to the circle $x^2 + y^2 + Ax + By + C = 0$ in a point on the circle</i></p> <p>11. Educators say that students will be able to answer this question</p>	<p>65 minutes</p>	<p>Step Think</p> <p>Step Talk</p>

<p>Phase 3: Organizing students into learning teams</p> <p>12. Educators form groups of 4-5 people and condition the students to sit in the group.</p> <p>13. Educators distribute Student Worksheet to all students</p> <p>14. Educators explain the steps that students will take in working on Student Worksheet</p> <p>Phase 4: Helping Teamwork and Learning to Experiment</p> <p>15. The educator asks students to observe activity 1 to determine the formula for the equation tangent to the circle $x^2 + y^2 + Ax + By + C = 0$ at some point on the circle and solve the problems given in the Student Worksheet individually and mark things that are not understood</p> <p>Reasoning</p> <p>16. Educators provide opportunities for students to communicate or discuss in their groups the results of their observations in activity 1 contained in Student Worksheet</p> <p>17. Educators ask students to work on questions in activity 1 in the Student Worksheet in groups</p> <p>Communicating</p> <p>18. Educators randomly select groups that will present the results of the discussion in front of the class</p> <p>19. Educators ask students to write down the conclusions obtained at the Think and Talk stage in the language they understand.</p> <p>Phase 5: Evaluating</p> <p>20. Educators provide confirmation and reinforcement of the results of discussion and presentation of students and provide additional material explanation if something is missing</p>		Step Write
<p>Closing Activities</p> <p>Phase 6: Providing Recognition or Award</p>	15 minutes	

<p>21. Educators give awards to the best group at this meeting</p> <p>22. The educator together with the students concludes the lessons for this meeting</p> <p><i>The equation tangents to the circle</i> $x^2 + y^2 + Ax + By + C = 0$ at a point on the circle ie $x_1x + y_1y + \frac{A}{2}(x + x_1) + \frac{B}{2}(y + y_1) + C = 0$.</p> <p>23. Educators give quizzes to students regarding the material they have just learned</p> <p>24. Educators give homework to students related to the material at this meeting in the mathematics perspective student book page 81 numbers 3,6 and 7</p> <p>25. The educator asks students to read the next material about the equation tangent to a circle at a point outside the circle in the mathematics perspective student book</p> <p>26. The educator closes the lesson by saying hamdalah</p>		
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The sixth meeting (2JP)

Description of Learning Activities	Allocation Time	Step TTW
<p>Preliminary activities</p> <ol style="list-style-type: none"> 1. Educators open the meeting by saying greetings 2. Educators together with students read prayers, Asma'ul Husna, and read the Koran (if during the first lesson) and check the attendance of students. <p>Phase 1: Conveying Objectives and Preparing Students</p> <ol style="list-style-type: none"> 3. Educator prepare participants students to study. 4. Educator explain procedure learning by using the cooperative model of the TTW (Think Talk Write) type: 	10 minutes	

<p>a. Students will discuss in groups that will be divided by the teacher.</p> <p>b. One of the groups will be selected to present the results of the discussion to the class.</p> <p>5. Educators inform the learning objectives and learning outcomes that are expected to be achieved.</p> <p>6. Apperception: Educators conduct questions and answers with students about material related to learning to be studied</p> <p><i>Do you still remember the equation tangent to the circle $x^2 + y^2 + Ax + By + C = 0$ in a point on the circle</i></p> <p>7. Motivation: Educators conduct questions and answers with students about the benefits of studying the material being studied</p> <p><i>A foreign fishing boat from China enters Indonesian Territory in Natuna with coordinates (3, - 1), the fishing boat also has a radar with a range of 5 km in all directions to find out other vessels in the vicinity. If a fishing boat is detected by an Indonesian patrol boat at coordinate (10,0) and wants to attack the fishing boat so that it doesn't escape, then the patrol boat path can be determined so that it is not detected by the fishing boat by studying the equation of the tangent to the circle at a point outside the circle</i></p>		
<p>Core activities</p> <p>Phase 2: Presenting Information</p> <p>8. Educators provide information and problems related to the equation tangent to a circle at a point outside the circle to students</p> <p><i>Determine the equation for the tangent to the circle $x^2 + y^2 = 25$ drawn from the point (10,0) outside the circle!</i></p> <p>Observe</p> <p>9. Students are asked to pay attention to the problems given by the educator</p> <p><i>Determine the equation for the tangent to the circle $x^2 + y^2 = 25$ drawn from the point (10,0) outside the circle!</i></p>	65 minutes	

Phase 5: Evaluating 20. Educators provide confirmation and reinforcement of the results of discussion and presentation of students and provide additional material explanation if anything is missing		
Closing Activities Phase 6: Providing Recognition or Award 21. Educators give awards to the best group at this meeting 22. The educator together with the students concludes the lessons for this meeting 23. Educators give quizzes to students regarding the material they have just learned 24. Educators give homework to students related to the material at this meeting in the mathematics perspective student book page 86 number 1-5 25. Educators ask students to repeat material that has been previously taught at home 26. The educator closes the lesson by saying hamdalah	15 minutes	

G. Assessment

The assessment is carried out during the learning activities, namely:

1. Knowledge Assessment: the final test

Arranged by

Vanya Aridanthy
NIM. 16029040

Appendix 7

VALIDATION SHEET OF STUDENT WORKSHEET

Education Unit : SMA Adabiah 2 Padang

Subjects : Mathematics

Class / Semester : XI / 2

Subject : Equations of Tangents to Circles

ASSESSMENT INSTRUCTIONS:

Put a check mark (✓) in the answer column below.

6) Format

No	Aspects to be Evaluated	Assessment		Note
		Yes	Not	
1	The distribution of material on the Student Worksheet is clear	✓		
2	The numbering system on the Student Worksheet is clear	✓		
3	The layout of the Student Worksheet is clear	✓		
4	The type and size of letters on the Student Worksheet is clear	✓		
5	The Student Worksheet used makes it easy for students to use it	✓		

7) Contents

No	Aspects to be Evaluated	Assessment		Note
		Yes	Not	
1	Student Worksheets according to learning objectives	✓		
2	The Student Worksheet contains instructions for use	✓		
3	The material on the Student Worksheet is in accordance with KI and KD	✓		
4	The activities contained in the Student Worksheet begin with providing contextual problems	✓		

	Contextual problems on the Student Worksheet help students in proposing conjectures from a statement	✓		
	Contextual problems on the Student Worksheet help students to draw conclusions from a statement	✓		
	Contextual problems in the Student Worksheet help students to provide alternatives to an argument	✓		

8) Question

No	Aspects to be Evaluated	Assessment		Note
		Yes	Not	
1	The questions on the Student Worksheet are in accordance with the objectives to be achieved	✓		
2	Questions on the Student Worksheet support conceptual understanding	✓		

9) Language

No	Aspects to be Evaluated	Assessment		Note
		Yes	Not	
1	Student Worksheets use language that is easy for students to understand	✓		
2	Student Worksheets use Indonesian that is good and correct	✓		
3	The sentences used are in accordance with the level of development of students	✓		
4	The sentences used are clear and simple	✓		

10) General Assessment

No	Aspects to be Evaluated	Assessment		Note
		Yes	Not	
1	Student Worksheets can be used without revision			
2	The Student Worksheet can be used with minor revisions	✓		
3	The Student Worksheet can be used with multiple revisions			
4	Student Worksheet cannot be used (still requires consultation)			

Comments and Suggestions

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Padang, February 2020

Validator

(Dr. Yerizon, M.Si)

MEETING STUDENT

WORKSHEET 1

Name:

Class :

Group :



SCORE	ATTITUDE

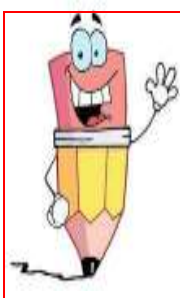
Basic competencies :
Analyze circles analytically

Indicators:
Determine the tangent to the circle
Analyze the equation for the tangent to the circle centered at point O (0,0) and the radius r if the slope is known

Purpose :

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation of the tangent to the circle centered at point O (0,0) and the radius r if the slope is known
2. Analyze the equation of the tangent of a circle centered at O (0,0) and of radius r if the gradient is known

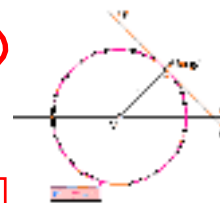
**INSTRUCTIONS:**

1. Fill in the identity in the space provided
2. Read carefully the following Student Worksheet
3. In this lesson there are 3 stages that each group must follow
 - a. *Think* : look at and write down the solutions to the problems given in Student Worksheet in activity 1: think in your own way
 - b. *Talk* : discuss with your group members about solving the problems that have been worked on at the think stage
 - c. *Write* : write down the results of the discussion with your group of friends on the paper provided
4. Ask the teacher for help if there are difficulties
5. Present the results of the discussion in front of the class

GA



Do you know what tangents to a circle are?



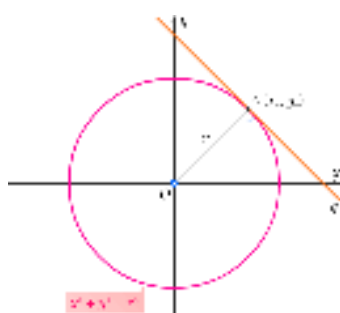
Look for information about the meaning of circle tangents and write your answer in the following column!

ACTIVITY 1 (Think)

After reading the material about the equation tangent to a circle centered at O (0,0) and the radius of r if the gradient is known, then look at problems 1 and 2 below and think about and write down the solutions to the problems given individually!



PROBLEM 1 (Rediscover the equation for the tangent to the circle centered at point O (0,0) and radius r if the slope is known)



Picture 1.

The image above is a circle \square centered at (0,0) and radius r with the equation: $x^2 + y^2 = r^2$ and the line g which pertains to circle L and has a gradient m: $y = mx + k$

We will determine the equation for the tangent to the circle centered at O (0,0) and the radius r if the slope is known by the following steps:

Substitute the equation for the line $g: y = mx + k$ to the circular equation $x^2 + y^2 = r^2$, and write it in the column below!



Did you get a quadratic equation with the variable? if yes, then we will do the next step.

Since line g is tangent to circle L , then line g and circle L have only one common point and have only one solution, namely the value $D = 0$



Try to remember again about Discriminant (D) and complete the solution above in the column below!

What is the value of k ?

Try to write the equation tangent to g in the column below!



PROBLEM 2 (applying the formula for the equation tangent to a circle centered at point O (0,0) and radius r if the slope is known)

To better understand the application of the formula obtained above, please solve the questions below!

Given that the equation for the circle $x^2 + y^2 = 16$ is centered at point O (0,0) and has a slope of $2\sqrt{2}$, find the equation of the tangent!

Solution:

ACTIVITY 2 (Talk)

Discuss the answers obtained in activity 1 with your group friends!



ACTIVITY 3 (Write)

Write a report on the results of the discussion and the conclusion of activity 2 on the provided sheet!



A large, empty rectangular box with a dashed red border, intended for writing a report.

**CHECK UNDERSTANDING**

Determine the equation of the tangent to the following circles with the gradient mentioned.

1. $L \equiv x^2 + y^2 = 18$ with slope -1 .
2. $L \equiv x^2 + y^2 = 20$ with gradient 2 .

A large, empty rectangular box with a dashed red border, intended for writing the solution to the check understanding questions.

MEETING STUDENT

WORKSHEET 2

Name:

Class :

Group :



SCORE	ATTITUDE

Basic competencies :

3.3 Analyze circles analytically

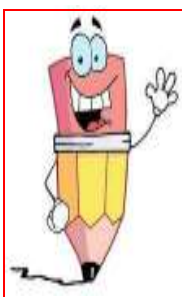
Indicators:

Analyze the equation of the tangent to the circle centered at point $P(a, b)$ and of radius r if the slope is known

Purpose :

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation for the tangent to the circle centered at point $P(a, b)$ and the radius r if the slope is known
2. Analyze the equation of the tangent to the circle centered at $P(a, b)$ and the radius r if the slope is known

**INSTRUCTIONS:**

1. Fill in the identity in the space provided
2. Read carefully the following Student Worksheet
3. In this lesson there are 3 stages that each group must follow
 - a. *Think* : look at and write down the solutions to the problems given in Student Worksheet in activity 1: think in your own way
 - b. *Talk* : discuss with your group members about solving the problems that have been worked on at the think stage
 - c. *write* : write down the results of the discussion with your group of friends on the paper provided
4. Ask the teacher for help if there are difficulties
5. Present the results of the discussion in front of the class

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ACTIVITY 1 (Think)

After reading the material about the equation tangent to a circle centered at $P(a, b)$ and the radius of r if the slope is known, then look at problems 1 and 2 below and think about and write down the solutions to the problems given individually!



PROBLEM 1 (Finding the equation for the tangent to the circle centered at point $P(a, b)$ and of radius r if the slope is known)

Remember again!

- Equation of a line with a gradient m namely $y = mx + n$
- Equation of the circle $L \equiv (x - a)^2 + (y - b)^2 = r^2$

We will determine the equation for the tangent to the circle centered at $P(a, b)$ and the radius r if the slope is known by the following steps:

Substitute the equation for the line with the slope m in the circular equation L !



Write the conditions for a line tangent to the circle!

After knowing terms one line offend circle, step next Find the value of n from the discriminant value of the line equation in the column below!

Try writing the equation of the tangent by substituting the value of n obtained for the line equation $y = mx + n$ in the column below!



PROBLEM 2 (applying the formula for the equation tangent to a circle centered at point $P(a, b)$ and of radius r if the slope is known)

To better understand the application of the formula obtained, please solve the questions below!

Determine the equation for the tangent to the circle $x^2 + y^2 - 6x + 10y - 46 = 0$ which has gradient 2!

Solution:

ACTIVITY 2 (Talk)

Discuss the answers obtained in activity 1 with your group friends!



ACTIVITY 3 (Write)

Write a report on the results of discussion on activity 2 on the provided sheet!



A large, empty rectangular area with rounded corners, outlined by a dashed red line, intended for writing a report.

**CHECK UNDERSTANDING**

Determine the equation of the tangent to the following circles with the gradient mentioned.

1. $L \equiv (x - 1)^2 + (y - 3)^2 = 25$ with gradient -2 .

2. $L \equiv x^2 + y^2 - 2x + 4y - 3 = 0$ with gradient 2 .

MEETING STUDENT

WORKSHEET 3

Name:

Class :

Group :



SCORE	ATTITUDE

Basic competencies :

3.3 Analyze circles analytically

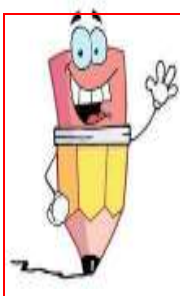
Indicators:

Analyze the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$

Purpose :

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$ exactly
2. Analyze the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$ exactly

**INSTRUCTIONS:**

1. Fill in the identity in the space provided
2. Read carefully the following Student Worksheet
3. In this lesson there are 3 stages that each group must follow
 - a. *Think* : look at and write down the solutions to the problems given in Student Worksheet in activity 1: think in your own way
 - b. *Talk* : discuss with your group members about solving the problems that have been worked on at the think stage
 - c. *write* : write down the results of the discussion with your group of friends on the paper provided
4. Ask the teacher for help if there are difficulties
5. Present the results of the discussion in front of the class

GA

ACTIVITY 1 (Think)

After reading the material about the equation tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$, after that, look at problems 1 and 2 below and think about and write down the solutions to the problems given individually!



PROBLEM 1 (Rediscovering the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 = r^2$)



Picture 1.

In the picture above, it can be seen that point A is on the circle $x^2 + y^2 = r^2$. Suppose the tangent line ℓ has a slope of m_2 , then m_2 is (determine the value of m_1 first):



Hence, the equation for the line ℓ which has the slope m_2 and passes through the point $A(x_1, y_1)$ is $y - y_1 = m_2(x - x_1)$

Try to write the equation tangent ℓ by substituting the value m_2 into the equation above in the column below!



PROBLEM 2 (applying the equation tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 =$)

To better understand the application
of the formula obtained, please
solve the questions below!

Find the equation for the tangent to the circle passing through point $(2,0)$ to center P $(0,0)$ and of radius 3!

Solution:

ACTIVITY 2 (Talk)

Discuss the answers obtained in activity 1 with your group friends!

**ACTIVITY 3 (Write)**

Write a report on the results of discussion on activity 2 on the provided sheet!



A large, empty rectangular box with a dashed red border, intended for writing a report.

**CHECK UNDERSTANDING**

Determine the equation of the tangent to the following circles!

1. $x^2 + y^2 = 4$ at point $(1, \sqrt{3})$

2. $x^2 + y^2 = 10$ at point $(-3, 1)$

A large, empty rectangular box with a dashed red border, intended for writing the solutions to the problems.

MEETING STUDENT

WORKSHEET 4

Name:

Class :

Group :



SCORE	ATTITUDE

Basic competencies :

3.3 Analyze circles analytically

Indicators:

Analyze the equation for the tangent to the circle at point A (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$

Purpose :

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation for the tangent to the circle at point A (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$ exactly
2. Analyze the equation for the tangent to the circle at point R (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$ exactly

**INSTRUCTIONS:**

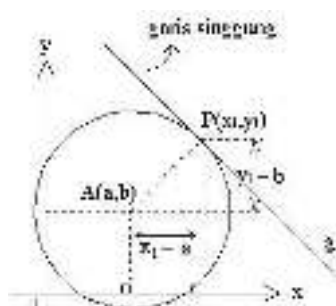
1. Fill in the identity in the space provided
2. Read carefully the following Student Worksheet
3. In this lesson there are 3 stages that each group must follow
 - a. *Think* : look at and write down the solutions to the problems given in Student Worksheet in activity 1: think in your own way
 - b. *Talk* : discuss with your group members about solving the problems that have been worked on at the think stage
 - c. *write* : write down the results of the discussion with your group of friends on the paper provided
4. Ask the teacher for help if there are difficulties
5. Present the results of the discussion in front of the class

ACTIVITY 1 (Think)

After reading the material about the equation tangent to the circle at point R (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$, then look at problems 1 and 2 below and think about and write down the solution to the problem given individually!



PROBLEM 1 (Rediscovering the equation for the tangent to the circle at point R (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$)



Picture 1.

Look at the image above and determine the gradient for the line AP (m_1) and also m_2 !

Thus, the equation for the line g which has a slope of m_2 and through the point (x_1, y_1) is (equation 1 is obtained)

Since point (x_1, y_1) lies on the circle, substitute point $P(x_1, y_1)$ for the circle.

$(x - a)^2 + (y - b)^2 = r^2$, we get:

Try substituting the form above to equation 1 and write it in the column below!

From the description above we can conclude as follows:



PROBLEM 2 (applying the equation tangent to the circle at point R ($P(x_1, y_1)$) on the circle $(x - a)^2 + (y - b)^2 = r^2$)

To better understand the application of the formula obtained, please solve the questions below!

Find the equation for the tangent of the circle through the point (2,4) with the equation of the circle is $(x - 1)^2 + (y - 2)^2 = 5$!

Solution:

ACTIVITY 2 (Talk)

Discuss the answers obtained in activity 1 with your group friends!

**ACTIVITY 3 (Write)**

Write a report on the results of discussion on activity 2 on the provided sheet!



A large, empty, rounded rectangular box with a dashed red border, intended for writing a report.

**CHECK UNDERSTANDING**

Determine the equation of the tangent to the following circles!

1. $(x - 1)^2 + (y - 2)^2 = 4$ at point A (-1,2)
2. $(x - 10)^2 + (y - 1)^2 = 61$ at point B (15, -5)

A large, empty rectangular box with rounded corners and a dashed red border, intended for the student to show their work.

MEETING STUDENT

WORKSHEET 5

Name:

Class :

Group :



SCORE	ATTITUDE

Basic competencies :

3.3 Analyze circles analytically

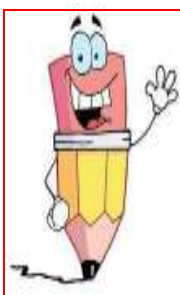
Indicators:

Analyze the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$

Purpose :

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Find the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$ exactly
2. Analyze with the equation the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$ dengan = 0 with precision

**INSTRUCTIONS:**

1. Fill in the identity in the space provided
2. Read carefully the following Student Worksheet
3. In this lesson there are 3 stages that each group must follow
 - a. *Think* : look at and write down the solutions to the problems given in Student Worksheet in activity 1: think in your own way
 - b. *Talk* : discuss with your group members about solving the problems that have been worked on at the think stage
 - c. *write* : write down the results of the discussion with your group of friends on the paper provided
4. Ask the teacher for help if there are difficulties
5. Present the results of the discussion in front of the class

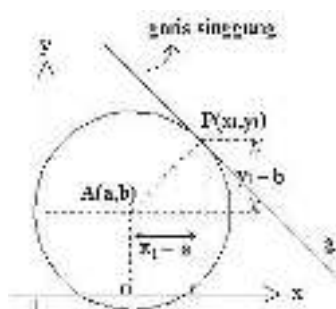
GA

ACTIVITY 1 (Think)

After reading the material about the equation tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$, then look at problems 1 and 2 below and think about and write down the solutions to the problems given individually !



PROBLEM 1 (Rediscovering the equation for the tangent to the circle at point A (x_1, y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$)



Picture 1.

Look at the picture above! To determine the equation for the tangent to the circle at point a (x_1, y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$. it is enough to describe the tangent to the circle at point R (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$ which has been studied before, and make your answer in the column below!

From the description above we can conclude as follows.



PROBLEM 2 (applies the formula for the equation tangent to the circle at point A (x_1 , y_1) on the circle $x^2 + y^2 + Ax + By + C = 0$)

To better understand the application
of the formula obtained, please
solve the questions below!

Find the equation for the tangent to the circle $x^2 + y^2 - 6x + 8y - 120 = 0$ yang
through the dots (15, -5)!

Solution:

ACTIVITY 2 (Talk)

Discuss the answers obtained in activity 1 with your group friends!

**ACTIVITY 3 (Write)**

Write a report on the results of discussion on activity 2 on the provided sheet!



A large, empty, rounded rectangular box with a dashed red border, intended for writing a report.

**CHECK UNDERSTANDING**

Determine the equation of the tangent to the following circles!

1. $x^2 + y^2 - 6x + 4y - 12 = 0$ at point A (7,1)
2. $x^2 + y^2 - 2x - y - 27 = 0$ at point B (3,1)

MEETING STUDENT

WORKSHEET 6

Name:

Class :

Group :



SCORE	ATTITUDE

Basic competencies :

3.3 Analyze circles analytically

Indicators:

Analyze the equation for the tangent to a circle
at a point outside the circle

Purpose :

Through learning activities using the Think Talk Write model, discussion methods, question and answer and assignments as well as with a scientific approach and working on student worksheet, students are expected to be able to:

1. Analyze the equation for the tangent to a circle at a point outside the circle precisely
2. Solve the problem of the equation tangent to the circle at a point outside the circle exactly

INSTRUCTIONS:

1. Fill in the identity in the space provided
2. Read carefully the following Student Worksheet
3. In this lesson there are 3 stages that each group must follow
 - a. *Think* : look at and write down the solutions to the problems given in Student Worksheet in activity 1: think in your own way
 - b. *Talk* : discuss with your group members about solving the problems that have been worked on at the think stage
 - c. *write* : write down the results of the discussion with your group of friends on the paper provided
4. Ask the teacher for help if there are difficulties
5. Present the results of the discussion in front of the class

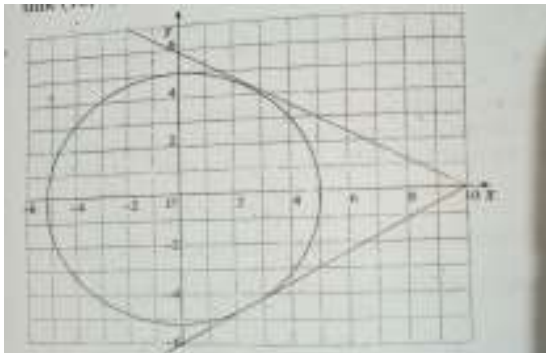
GA

ACTIVITY 1 (Think)

After reading the material about the equation tangent to a circle at a point outside the circle, after that look at problems 1 and 2 below and think about and write down the solutions to the problems given individually!



PROBLEM 1 (Applying the formula for the equation tangent to a circle at a point outside the circle exactly)



Look at the picture above!

Find the equation for the tangent to the circle $x^2 + y^2 = 25$ drawn from the point $(10,0)$ outside the circle!

Suppose the slope of the tangent m and through the point $(10,0)$ outside the circle.

$$y = mx - mx_1 + y_1 \leftrightarrow y = mx - (10) + 0 \leftrightarrow y = mx - 10m.$$

Step 1:

Find the slope of the tangent m and through the point $(10,0)$ outside the circle then substitute y for the equation of the circle, write your answer in the column below!

Step 2:

Calculate the discriminant value D in the column below!

Step 3:

By substituting m_1 and m_2 to $y = mx - 10m$, the equation of the tangent line is obtained.



PROBLEM 2 (solve problems regarding the equation of tangents to a circle at a point outside the circle exactly)

To better understand the problems above, please solve the questions below!

Find the equation for the tangent to the circle $x^2 + y^2 = 25$ drawn from the point $(10,0)$ outside the circle!

Solution:

ACTIVITY 2 (Talk)

Discuss the answers obtained in activity 1 with your group friends!

ACTIVITY 3 (Write)

Write a report on the results of discussion on activity 2 on the provided sheet!





CHECK UNDERSTANDING

Determine the equation of the tangent to the following circles!

1. $x^2 + y^2 = 225$ drawn from the point (15, -5)
2. $x^2 + y^2 = 25$ drawn from the point (7,1)

A large, empty rounded rectangle with a dashed red border, intended for the student to show their work.

Appendix 9

QUIZ QUESTION GRID

Education units : SMA Adabiah 2 Padang
 Subjects : Mathematics
 Class / Semester : XI / II
 Theory : Equations Tangent Circle
 Time Allocation : 15 Minute

Competence Basic	Indicator			Question	Cognitive Level	Item
	Indicator Achievement Competence	Indicator Understanding Concept	Problem Indicators			
3.3	Analyze tangent equation that circle centered on a point O (0,0) and radius r if known the gradient	Classify an objects based on fulfilled not requirements that forming the concept	Participants students could determine equation line tangent to circle	Check whether the equation is tangent to the circle $x^2 + y^2 = 20$ and graded 2 is $y = 2x \pm 10$!	C3	1
	Analyze line equations tangent circle that centered on a point P (a, b) and radius r if is known the gradient	Classify objects based on fulfilled not requirements that forming the concept	Participants students can determine the equation tangents to the circle	Check whether the equation is tangent to the circle and gradient 2 is !	C3	2

Analyze tangent equation circle in point A (x_1, y_1) on circle $x^2 + y^2 = r^2$	Identify traits operation or concept	Given a circular equation through a point, students can identify the point located on the circle or outside the circle, and determine the line equation the point	Check whether the equation for the tangent of the circle through the point (-4,3) to the circle $x^2 + y^2 = 25$ is $3y - 4x - 25 = 0$!	C3	3
Analyze equation of the line tangent circle on point A (x_1, y_1) on the circle $(x - a)^2 + (y - b)^2 = r^2$	Identify traits operation or concept	Given a equation that circle through a point, learners could identify that point lies on circle or in outside the circle, and determine equation of the line the point	Check whether the equation is tangent to the circle which extends the point (2,4) to the circle is $(x - 1)^2 + (y - 2)^2 = 5$ $x + 2y = 0$	C3	4

	<p>Analyze tangent equation in circle in point A (x_1, y_1) on circle</p> $x^2 + y^2 + Ax + By + C = 0$	<p>Identify operating properties or concept</p>	<p>Granted a circular equation that through a point, learners could identify the point located on circle or in outside the circle, and determine equation line the point</p>	<p>Check whether the equation for the tangent of the circle which extends the point (7,1) to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is $4x + 3y - 31 = 0$!</p>	C3	5
	<p>Analyze equation of the line tangent circle in something point in outside the circle</p>	<p>Develop terms necessary or sufficient terms of a concept</p>	<p>Given a equation circle with a point which is offend the circle, students can determine wrong one point the point</p>	<p>Check whether the equation is tangent to the circle $x^2 - 2x + y^2 + 4y = 0$ that through point A (0,1) is $y_1 = 2x + 1$ and $y_2 = 2x - 9$!</p>	C3	6

Appendix 10

RUBRIC SCORING QUIZ OF CIRCLE EQUATION MATERIALS

Indicator Understanding n Concept	Question	Answer
MEETING I		
Classify objects based on them whether the requirements are fulfilled or not that shape k concept the	Check whether the equation is tangent to the circle $x^2 + y^2 = 20$ and gradient 2 is $y = 2x \pm 10$!	Score 0 No answer
		Score 1 Is known: $x^2 + y^2 = 20$ $m = 2$ Wanted: y? Answer:
		Score 2 Is known: $x^2 + y^2 = 20$ $m = 2$ Wanted: y? Answer: Circle $x^2 + y^2 = 20$ have center point (0,0) and radius $r = \sqrt{20} = 2\sqrt{5}$ $y = mx \pm r\sqrt{1+m^2}$
		Score 3 Is known: $x^2 + y^2 = 20$ $m = 2$ Wanted: y? Answer: Circle $x^2 + y^2 = 20$ have center point (0,0) and radius $r = \sqrt{20} = 2\sqrt{5}$ $y = mx \pm r\sqrt{1+m^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{1+2^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{5}$ $y = 2x \pm 10$
		Score 4 Is known:

		$x^2 + y^2 = 20$ $m = 2$ Asked: $y?$ Answer: Circle $x^2 + y^2 = 20$ have center point $(0,0)$ and radius $r = \sqrt{20} = 2\sqrt{5}$ $y = mx \pm r\sqrt{1+m^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{1+2^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{5}$ $y = 2x \pm 10$ So it is proved that the equation is tangent to the circle $x^2 + y^2 = 10$ with gradient 2 is $y = 2x + 10$ or $y = 2x - 10$
MEETING II		
Classify objects based on them whether the requirements are fulfilled or not that shape concept the	Check what the equation is tangent to circle $x^2 + y^2 - 2x + 4y - 3 = 0$ and is graded 2 $y = 2x - 4 - 2\sqrt{10}!$	Score 0 No answer
		Score 1 Is known: $x^2 + y^2 - 2x + 4y - 3 = 0$ $m = 2$ Wanted: $y?$ Answer:
		Score 2 Is known: $x^2 + y^2 - 2x + 4y - 3 = 0$ $m = 2$ Wanted: $y?$ Answer: Circle $x^2 + y^2 = 20$ have center point $(0,0)$ and Fingers: $r = \sqrt{1^2 + (-2)^2 - (-3)}$ $= \sqrt{1+4+3}$ $= \sqrt{8} = 2\sqrt{2}$ $y - b = m(x - a) \pm r\sqrt{1+m^2}$
		Score 3 Is known: $x^2 + y^2 - 2x + 4y - 3 = 0$ $m = 2$ Wanted: $y?$ Answer: Circle $x^2 + y^2 = 20$ have center point $(0,0)$ and Fingers: $r = \sqrt{1^2 + (-2)^2 - (-3)}$

		$= \sqrt{1+4+3}$ $= \sqrt{8} = 2\sqrt{2}$ $y - b = m(x - a) \pm r\sqrt{1+m^2}$ $y + 2 = 2(x - 1) \pm 2\sqrt{2} \times \sqrt{1+2^2}$ $y + 2 = 2x - 2 \pm 2\sqrt{2} \times \sqrt{5}$ $y = 2x - 4 \pm 2\sqrt{10}$
		<p>Score 4</p> <p>Is known:</p> $x^2 + y^2 - 2x + 4y - 3 = 0$ $m = 2$ <p>Wanted: y?</p> <p>Answer:</p> <p>Circle $x^2 + y^2 = 20$ have center point $(0,0)$ and</p> <p>Fingers: $r = \sqrt{1^2 + (-2)^2 - (-3)}$</p> $= \sqrt{1+4+3}$ $= \sqrt{8} = 2\sqrt{2}$ $y - b = m(x - a) \pm r\sqrt{1+m^2}$ $y + 2 = 2(x - 1) \pm 2\sqrt{2} \times \sqrt{1+2^2}$ $y + 2 = 2x - 2 \pm 2\sqrt{2} \times \sqrt{5}$ $y = 2x - 4 \pm 2\sqrt{10}$ <p>So it is proven that the equation is tangent to circle $x^2 + y^2 - 2x + 4y - 3 = 0$ with gradient 2 is $y = 2x - 4 + 2\sqrt{10}$ Or $y = 2x - 4 - 2\sqrt{10}$</p>
MEETING III		
Identify the properties of the operation or concept.	Check whether the equation is tangent to the circle which extends the point $(-4,3)$ to the circle $x^2 + y^2 = 25$ is $3y-4x-25=0$!	<p>Score 0</p> <p>No answer</p>
		<p>Score 1</p> <p>Is known:</p> $x^2 + y^2 = 25$ <p>through point $(-4,3)$ Wanted: y?</p> <p>Answer:</p>
		<p>Score 2</p> <p>Is known:</p> $x^2 + y^2 = 25$ <p>through point $(-4,3)$ Wanted: y?</p>

		<p>Answer:</p> <p>Substitute it point (-4,3) to circle equation:</p> $(-4)^2 + 3^2 = 25$ $16 + 9 = 25$ $25 = 25$ <p>$r^2 = 25$, then point (-4,3) lies on the circle</p>
		<p>Score 3</p> <p>Is known: $x^2 + y^2 = 25$ through point (-4,3) Wanted: y?</p> <p>Answer:</p> <p>Substitute it point (-4,3) to circle equation:</p> $(-4)^2 + 3^2 = 25$ $16 + 9 = 25$ $25 = 25$ <p>$r^2 = 25$, then the point (-4,3) lies on the circle, then</p> $x_1x + y_1y = r^2$ $\Leftrightarrow -4x + 3y = 25$ $\Leftrightarrow 3y - 4x - 25 = 0$
		<p>Score 4</p> <p>Is known: $x^2 + y^2 = 25$ through point (-4,3) Wanted: y?</p> <p>Answer:</p> <p>Substitute it point (-4,3) to</p>

		<p>circle equation:</p> $(-4)^2 + 3^2 = 25$ $16 + 9 = 25$ $25 = 25$ <p>$r^2 = 25$, then the point $(-4,3)$ lies on the circle, then</p> $x_1x + y_1y = r^2$ $\Leftrightarrow -4x + 3y = 25$ $\Leftrightarrow 3y - 4x - 25 = 0$ <p>So it is proved that the equation of the tangent to the circle passing through the point $(-4,3)$ is $3y - 4x - 25 = 0$.</p>
MEETING IV		
Identify the properties of the operation or concept.	Check whether the equation is tangent to the circle which extends the point $(2,4)$ to the circle $(x-1)^2 + (y-2)^2 = 5$ is $x + 2y = 0$!	<p>Score 0 No answer</p>
		<p>Score 1 Is known: $(x-1)^2 + (y-2)^2 = 5$ through point $(2,4)$ Wanted: $y?$ Answer:</p>
		<p>Score 2 Is known: $(x-1)^2 + (y-2)^2 = 5$ through point $(2,4)$ Wanted: $y?$ Answer:</p> <p>Substitute it point $(2,4)$ to circle equation:</p> $(2-1)^2 + (4-2)^2 = 5$ $1^2 + 2^2 = 5$ $5 = 5$

		$r^2 = 5$, then point (2,4) lies on the circle
		Score 3 Is known: $(x - 1)^2 + (y - 2)^2 = 5$ through point (2,4) Wanted: y ? Answer: Substitute it point (2,4) to circle equation: $(2 - 1)^2 + (4 - 2)^2 = 5$ $1^2 + 2^2 = 5$ $5 = 5$ $r^2 = 5$, then point (2,4) lies on the circle, then $(x - a)(x_1 - a) + (y - b)(y_1 - b) = r^2$ $\Leftrightarrow (x - 1)(x_1 - 1) + (y - 2)(y_1 - 2) = 5$ $\Leftrightarrow (x - 1)(2 - 1) + (y - 2)(4 - 2) = 5$ $\Leftrightarrow (x - 1)1 + (y - 2)2 = 5$ $\Leftrightarrow x - 1 + 2y - 4 = 5$ $\Leftrightarrow x + 2y = 10$
		Score 4 Is known: $(x - 1)^2 + (y - 2)^2 = 5$ through point (2,4) Wanted: y ? Answer: Substitute it point (2,4) to circle equation: $(2 - 1)^2 + (4 - 2)^2 = 5$ $1^2 + 2^2 = 5$

		$5 = 5$ $r^2 = 5$, then point (2,4) lies on the circle, then $(x - a)(x_1 - a) + (y - b)(y_1 - b) = r^2$ $\leftrightarrow (x - 1)(x_1 - 1) + (y - b)(y_1 - b) = 5$ $\leftrightarrow (x - 1)(2 - 1) + (y - 2)(4 - 2) = 5$ $\leftrightarrow (x - 1)1 + (y - 2)2 = 5$ $\leftrightarrow x - 1 + 2y - 4 = 5$ $\leftrightarrow x + 2y = 0$ So it is proved that the equation is tangent to the circle through the point (2,4) is $x + 2y = 0$.
MEETING V		
Identify the properties of the operation or concept.	Check whether the equation of the tangent to the circle passing through the point (7,1) to circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is $4x + 3y - 31 = 0$!	Score 0 No answer
		Score 1 Is known: $x^2 + y^2 - 6x + 4y - 12 = 0$ through point (7,1) Wanted: $y?$ Answer:
		Score 2 Is known: $x^2 + y^2 - 6x + 4y - 12 = 0$ through point (7,1) Wanted: $y?$ Answer: Substitute it point (7,1) to circle equation: $7^2 + 1^2 - 6(7) + 4(1) - 12 = 0$ $49 + 1 - 42 + 4 - 12 = 0$ $0 = 0$ Then point (7,1) located on

		circle
		<p>Score 3</p> <p>Is known: $x^2 + y^2 - 6x + 4y - 12 = 0$ through point (7,1) Wanted: $y?$ Answer:</p> <p>Substitute it point (7,1) to circle equation:</p> $7^2 + 1^2 - 6(7) + 4(1) - 12 = 0$ $49 + 1 - 42 + 4 - 12 = 0$ $0 = 0$ <p>Then point (7,1) located on circle, then</p> $x_1x + y_1y + \frac{A}{2}(x + x_1) + \frac{B}{2}(y + y_1) = 0$ $\Leftrightarrow 7x + y - 6 + \frac{A}{2}(x + 7) + \frac{B}{2}(y + 1) - 12 = 0$ $\Leftrightarrow 7x + y - 21 - 3x + 2 + 2y - 12 = 0$ $\Leftrightarrow 4x + 3y - 31 = 0$
		<p>Score 4</p> <p>Dikeknow: $x^2 + y^2 - 6x + 4y - 12 = 0$ through point (7,1) Wanted: $y?$ Answer:</p> <p>Substitute it point (7,1) to circle equation:</p> $7^2 + 1^2 - 6(7) + 4(1) - 12 = 0$ $49 + 1 - 42 + 4 - 12 = 0$ $0 = 0$ <p>Then point (7,1) located on</p>

		<p>circle, then</p> $x_1x + y_1y + \frac{A}{2}(x + x_1) + \frac{B}{2}(y + y_1) = 0$ $\Leftrightarrow 7x + y - 6 + \frac{A}{2}(x + 7) + \frac{B}{2}(y + 1) - 12 = 0$ $\Leftrightarrow 7x + y - 21 - 3x + 2 + 2y - 12 = 0$ $\Leftrightarrow 4x + 3y - 31 = 0$ <p>So it is proved that the equation is tangent to the circle through the point (7,1) is $4x + 3y - 31 = 0$</p>
MEETING VI		
Develop the necessary or sufficient conditions of a concept	<p>Check what the equation is tangent to circle</p> $x^2 - 2x + y^2 + 4y = 0$ <p>that through point A (0,1) is $y_1 = 2x + 1$ and $y_2 = 2x - 9$!</p>	<p>Score 0</p> <p>No answer</p>
		<p>Score 1</p> <p>Is known: $x^2 - 2x + y^2 + 4y = 0$ through point A (0,1) Wanted: Answer:</p>
		<p>Score 2</p> <p>Is known: $x^2 - 2x + y^2 + 4y = 0$ through point A (0,1) Wanted: y? Answer:</p> <p>Substitute it point (0.1) to circle equation:</p> $0^2 - 2(0) + 1^2 + 4(1) = 0$ $\Leftrightarrow 0 - 0 + 1 + 4 = 0$ $\Leftrightarrow 5 = 0$ $\Leftrightarrow 5 > 0$
		<p>Score 3</p> <p>is known: $x^2 - 2x + y^2 + 4y = 0$ through point A (0,1) Wanted: y?</p>

		<p>Answer:</p> <p>Substitute a point (0,1) to the equation of the circle:</p> $0^2 - 2(0) + 1^2 + 4(1) \dots 0$ $\Leftrightarrow 0 - 0 + 1 + 4 \dots 0$ $\Leftrightarrow 5 \dots 0$ $\Leftrightarrow 5 > 0$ <p>then point (0,1) lies outside the circle</p> <p>where the center of the circle (1, -2) and $r = \sqrt{5}$, then the equation of the tangent to the circle is</p> $y - b = m(x - a) \pm r\sqrt{1 + m^2}$ $\Leftrightarrow y + 2 = m(x - 1) \pm \sqrt{5}\sqrt{1 + 5m^2}$ $\Leftrightarrow 1 + 2 = m(0 - 1) \pm \sqrt{5 + 5m^2}$ $\Leftrightarrow 3 = -m \pm \sqrt{5 + 5m^2}$ $\Leftrightarrow m + 3 = +\sqrt{5 + 5m^2}$ $\Leftrightarrow (m + 3)^2 = 5 + 5m^2$ $\Leftrightarrow m^2 + 6m + 9 - 5m^2 - 5 = 0$ $\Leftrightarrow 4m^2 - 6m - 4 = 0$ $\Leftrightarrow 2m^2 - 3m - 2 = 0$ $\Leftrightarrow (2m + 1)(m - 2) = 0$ $m = -\frac{1}{2} \text{ or } m = 2$ <p>So the tangent equation becomes</p>
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	$y + 2 = m(x - 1) \pm \sqrt{5}\sqrt{1 + 5m^2}$ $\Leftrightarrow y = 2(x - 1) - 2 \pm \sqrt{5}\sqrt{1 + 4}$ $\Leftrightarrow y = 2x - 4 \pm 5$
	<p>Score 4</p> <p>is known: $x^2 - 2x + y^2 + 4y = 0$ through point A (0,1) Wanted: y? Answer:</p> <p>Substitute a point (0,1) to the equation of the circle:</p> $0^2 - 2(0) + 1^2 + 4(1) \dots 0$ $\Leftrightarrow 0 - 0 + 1 + 4 \dots 0$ $\Leftrightarrow 5 \dots 0$ $\Leftrightarrow 5 > 0$ <p>then point (0,1) lies outside the circle</p> <p>where the center of the circle (1, -2) and $r = \sqrt{5}$, then the equation of the tangent to the circle is</p> $y - b = m(x - a) \pm r\sqrt{1 + m^2}$ $\Leftrightarrow y + 2 = m(x - 1) \pm \sqrt{5}\sqrt{1 + 5m^2}$ $\Leftrightarrow 1 + 2 = m(0 - 1) \pm \sqrt{5 + 5m^2}$ $\Leftrightarrow 3 = -m \pm \sqrt{5 + 5m^2}$ $\Leftrightarrow m + 3 = +\sqrt{5 + 5m^2}$ $\Leftrightarrow (m + 3)^2 = 5 + 5m^2$ $\Leftrightarrow m^2 + 6m + 9 - 5m^2 - 5 = 0$ $\Leftrightarrow 4m^2 - 6m - 4 = 0$

	$\Leftrightarrow 2m^2 - 3m - 2 = 0$ $\Leftrightarrow (2m + 1)(m - 2) = 0$ $m = -\frac{1}{2} \text{ or } m = 2$ <p>So the tangent equation becomes</p> $y + 2 = m(x - 1) \pm \sqrt{5}\sqrt{1 + 5m^2}$ $\Leftrightarrow y = 2(x - 1) - 2 \pm \sqrt{5}\sqrt{1 + 4}$ $\Leftrightarrow y = 2x - 4 \pm 5$ <p>So it is proven that the equation of the tangent is $y_1 = 2x + 1$ and $y_2 = 2x - 9$</p>
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Appendix 11**DISTRIBUTION OF EXPERIMENT CLASS QUIZ VALUE**

No. Sort Quiz Ke-	SCORE					
	I	II	III	IV	V	VI
1	50	25	50	100	100	-
2	25	50	50	50	50	100
3	25	100	50	75	50	50
4	25	25	50	-	50	100
5	25	25	50	50	50	50
6	25	50	50	50	100	100
7	-	50	75	100	100	100
8	25	50	50	50	100	50
9	25	50	75	50	50	50
10	50	75	100	100	75	50
11	50	50	75	50	50	50
12	25	25	50	50	50	100
13	25	25	50	75	100	100
14	25	25	-	50	50	50
15	25	25	-	50	75	50
16	50	75	75	75	100	100
17	25	50	100	75	50	100
18	25	50	50	50	50	75
19	50	50	100	100	50	50
20	25	50	50	50	100	100
21	25	50	50	50	50	50
22	25	50	50	50	100	100

23	25	50	50	50	50	50
total	675	1075	1300	1400	1550	1625
Lots Students	22	23	21	22	23	22
Average - Flat	30.68	46.74	61.90	63.63	67.39	73.86
The highest	50	100	100	100	100	100
Lowest	25	25	50	50	50	50
Value \geq 75	0	3	7	8	10	11
Value <75	22	20	14	14	13	11
% Completed	0.00	13.04	33.33	36.36	43.47	50.00
% Not Completed	100.00	86.96	66.67	63.64	56.53	50.00

Appendix 12

DISTRIBUTION OF EXPERIMENT CLASS QUIZ SCORE

No. Sort	SCORE					
Quiz Ke-Indicator	I	II	III	IV	V	VI
Problem To-1	2	2	3	3	3	8
2	1	1	1	1	1	1
3	2	1	2	4	4	-
4	1	2	2	2	2	4
5	1	4	2	3	2	2
6	1	1	2	-	2	4
7	1	1	2	2	2	2
8	1	2	2	2	4	4
9	1	2	2	2	4	4
10	-	2	3	4	4	4
11	1	2	2	2	4	2
12	1	2	3	2	2	2
13	2	3	4	4	3	2
14	2	2	3	2	2	2
15	1	1	2	2	2	4
16	1	1	2	3	4	4
17	1	1	-	2	2	2
18	1	1	-	2	3	2
19	2	3	3	3	4	4
20	1	2	4	3	2	4
21	1	2	2	2	2	3
22	2	2	4	4	2	2
	1	2	2	2	4	4

23	1		2		2		2		2		2	
total	27		44		52		56		62		57	
Lots Students	22		23		21		22		23		22	
Average	1.22		1.91		2.47		2.54		2.69		2.59	
The highest	2		3		4		4		4		4	
Lowest	1		1		2		2		2		2	
	F	%	F	%	F	%	F	%	F	%	F	%
Score 4	0	0.00	1	4.34	3	14.28	4	18.18	8	34.78	10	45.45
Score 3	0	0.00	2	8.69	4	19.04	4	18.18	2	8.69	1	0.045
Score 2	5	22.72	13	56.52	14	66.68	14	63.64	13	56.52	11	0.5
Score 1	17	77.28	7	30.44	0	0.00	0	0.00	0	0.00	0	0.00
Score 0	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00
Able	0	0.00	3	13.04	7	33.32	8	36.36	10	43.48	10	45.45
Less Able	22	100	20	86.96	14	66.68	14	63.64	13	56.52	12	0.5
Not Able	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00

Appendix 13

TEST TRIAL PROBLEMS GRATTERS TRY THE FINAL TEST

Education units : SMA Adabiah 2 Padang
 Class / Semester : XI / II
 Subjects : Math
 Subject matter : Equation Tangent to Circle
 Time Allocation : 2 JP (2 x 45 minutes)

Basic competencies :

3.3 Analyze circles analytically

Resolves problems associated with circles

No.	Concept Understanding Indicators	Problem Indicators	Question Number	Cognitive Level					
				C1	C2	C3	C4	C5	C6
1	Restate learned concepts.	Students can explain the meaning of a tangent to a circle	1	√					
2	Classify objects based on whether the requirements are met or not that form the concept.	Participants students can define equations tangent to circle	2			√			
3	Identify the properties of the operation or concept.	Given a circular equation through a point, students can identify the point located on the circle or outside the circle, and determine the equation of the tangent line	3				√		

4	Apply concepts logically.	Given a problem, students can determine the solution with use the equation tangent to the circle	4			√			
5	Give examples or examples counter (not example from	Given a circle image with a line that offends the circle, students can provide related information from the image	5		√				
6	Hook up various in math and outside	Given a problem regarding the equation of the line tangent to the circle, students can analyze the circle and solve the problem	6				√		
7	Develop the necessary or sufficient conditions of a concept	Given a circular equation with an abscissa point, students can determine the equation of the tangent to the circle	7				√		

Appendix 14

PROBLEMS FOR UNDERSTANDING THE CONCEPT OF MATHEMATICS

Education units	: SMA Adabiah 2 Padang
Class / Semester	: XI / II
Subjects	: Math
Subject matter	: Equation Tangent to Circle
Time Allocation	: 2 JP (2 x 45 minutes)

Instructions:

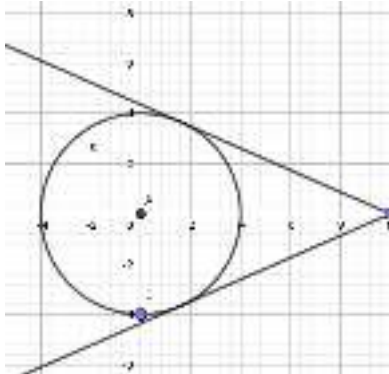
1. Pray first before working on the problem
 2. Write the name and class first on your answer sheet
 3. Read the questions carefully before answering them
 4. First, answer the questions that you think are easy
 5. Double-check your answer sheet before submitting it to the teacher.
-

1. Explain what is meant by tangents to circles!
2. Determine the equation of the tangent from circle $x^2 + y^2 = 20$ and graded 2!
3. Check whether the equation is tangent to the circle $(x - 10)^2 + (y - 1)^2 = 61$ through point (15, -5) ie $5x - 6y = 105$ by first identifying where the point is located on or outside the circle!
4. Specify equation line tangent from circle $x^2 + y^2 - 6x - 4y - 12 = 0$ which is perpendicular to the line $y = x + 4$!
- 5.



Mention the information contained in the circle above!

6.



Check whether the equation is tangent to the circle $x^2 + y^2 = 20$ that drawn from a point (10,0) outside the circle i.e. $2x + 4y = 20$!

7.

Circle $x^2 + y^2 - 2x + 13y + 40 = 0$

intersects the Y axis at $y = -5$. Check whether the equation of the tangent is $3y - 2x + 15 = 0$!

Appendix 15

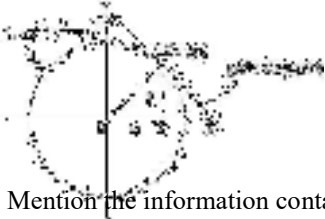
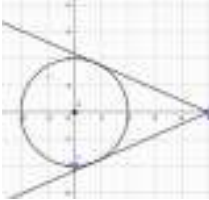
RUBIK SCORING TEST OF UNDERSTANDING MATHEMATIC CONCEPTS

NO	Test Questions	Answers to Questions	Score
1.	Explain what is meant by a tangent to the circle	No answer	0
		Tangents to the circle are lines that are offending circle	1
		Tangents to the circle are lines that are offends the circle at a point on the circle	2
		Tangents to the circle are lines that are pertains to the circle at one point and is perpendicular to the radius of the circle.	3
2	Determine the equation for the tangent to the circle $x^2 + y^2 = 20$ and graded 2	No answer	0
		Is known: $x^2 + y^2 = 20$ $m = 2$ Wanted: y? Answer:	1
		Is known: $x^2 + y^2 = 20$ $m = 2$ Wanted: y? Answer:	2

		Circle $x^2 + y^2 = 20$ has a center point (0,0) and radius $r = \sqrt{20} = 2\sqrt{5}$ $y = mx \pm r\sqrt{1+m^2}$	
		Is known: $x^2 + y^2 = 20$ $m = 2$ Wanted: y ? Answer: Circle $x^2 + y^2 = 20$ has a center point (0,0) and radius $r = \sqrt{20} = 2\sqrt{5}$ $y = mx \pm r\sqrt{1+m^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{1+2^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{5}$ $y = 2x \pm 10$	3
		Is known: $x^2 + y^2 = 20$ $m = 2$ Asked: y ? Answer: Circle $x^2 + y^2 = 20$ has a center point (0,0) and radius $r = \sqrt{20} = 2\sqrt{5}$ $y = mx \pm r\sqrt{1+m^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{1+2^2}$ $y = 2x \pm 2\sqrt{5} \times \sqrt{5}$ $y = 2x \pm 10$ So the equation is tangent to the circle $x^2 + y^2 = 10$ with gradient 2 is $y = 2x + 10$ or $y = 2x - 10$	4
3	Check whether the equation is tangent to the circle $(x-10)^2 + (y-1)^2 = 61$ that through dots (15, -5) ie $5x - 6y = 105$ by first identifying where the point is located on or outside the circle!	No answer	0
		Is known: $(x-10)^2 + (y-1)^2 = 61$ through dots (15, -5) Wanted: y ? Answer:	1
		Is known: $(x-10)^2 + (y-1)^2 = 61$ through dots (15, -5) Wanted: y ? Answer: Substitute the point (15, -5) into the equation for the circle: $(15-10)^2 + (-5-1)^2 = 25 + 36 = 61$ $r^2 = 61$, then point (15, -5) lies at	2

		circle	
		<p>Is known: $(x - 10)^2 + (y - 1)^2 = 61$ through dots (15, -5) Wanted: y?</p> <p>Answer:</p> <p>Substitute the point (15, -5) into the equation for the circle:</p> $(15 - 10)^2 + (-5 - 1)^2 = 25 + 36 = 61$ <p>$r^2 = 61$, then the point (15, -5) lies on the circle, then</p> $(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$ $\Leftrightarrow (15 - 10)(x - 10) + (-5 - 1)(y - 1) = 61$ $\Leftrightarrow 5(x - 10) + (-6)(y - 1) = 61$ $\Leftrightarrow 5x - 50 - 6y + 6 = 61$ $\Leftrightarrow 5x - 6y = 105$	3
		<p>Is known: $(x - 10)^2 + (y - 1)^2 = 61$ through dots (15, -5) Wanted: y?</p> <p>Answer:</p> <p>Substitute the point (15, -5) into the equation for the circle:</p> $(15 - 10)^2 + (-5 - 1)^2 = 25 + 36 = 61$ <p>$r^2 = 61$, then the point (15, -5) lies on the circle, then</p> $(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2$ $\Leftrightarrow (15 - 10)(x - 10) + (-5 - 1)(y - 1) = 61$ $\Leftrightarrow 5(x - 10) + (-6)(y - 1) = 61$ $\Leftrightarrow 5x - 50 - 6y + 6 = 61$	4

		$\leftrightarrow 5x - 6y = 105$ So the equation proves that the tangent to the circle through the point (15, -5) is $5x - 6y = 105$	
4	Determine the equation of the tangent circle $x^2 + y^2 - 6x - 4y - 12 = 0$ that perpendicular to the line $y = x + 4$!	No answer	0
		Is known : $x^2 + y^2 - 6x - 4y - 12 = 0$ Perpendicular line $y = x + 4$ Asked: y_2 ? Answer:	1
		Is known : $x^2 + y^2 - 6x - 4y - 12 = 0$ Perpendicular line $y = x + 4$ Asked: y_2 ? Answer: $y = x + 4$ then $m_1 = 1$ $y \perp y_2$, then $m_2 = -1$ Center Circle (3,2) and $r = 5$	2
		Is known : $x^2 + y^2 - 6x - 4y - 12 = 0$ Perpendicular line $y = x + 4$ Asked: y_2 ? Answer: $y = x + 4$ then $m_1 = 1$ $y \perp y_2$, then $m_2 = -1$ Center Circle (3,2) and $r = 5$ Then the equation of the tangent $y - b = m(x - a) \pm r\sqrt{1 + m^2}$ $\leftrightarrow y - 2 = -1(x - 3) \pm 5\sqrt{1 + (-1)^2}$ $\leftrightarrow y - 2 = -x + 3 \pm 5\sqrt{2}$ $\leftrightarrow y = -x + 5 \pm 5\sqrt{2}$	3
		Is known : $x^2 + y^2 - 6x - 4y - 12 = 0$ Perpendicular line $y = x + 4$ Asked: y_2 ? Answer: $y = x + 4$ then $m_1 = 1$ $y \perp y_2$, then $m_2 = -1$ Center Circle (3,2) and $r = 5$ Then the equation of the tangent $y - b = m(x - a) \pm r\sqrt{1 + m^2}$ $\leftrightarrow y - 2 = -1(x - 3) \pm 5\sqrt{1 + (-1)^2}$ $\leftrightarrow y - 2 = -x + 3 \pm 5\sqrt{2}$ $\leftrightarrow y = -x + 5 \pm 5\sqrt{2}$ So the equation for the tangent to the circle which is perpendicular to the line $y = x + 4$ is $y = -x + 5 + 5\sqrt{2}$ and $y = -x + 5 - 5\sqrt{2}$	4

5	 <p>Mention the information contained in the circle above!</p>	No answer	0
		Circle equation $L \equiv x^2 + y^2 = r^2$	1
		a. Equality n Circle $L \equiv x^2 + y^2 = r^2$ b. Tangent perpendicular to the line OP	2
		a. Equality n Circle $L \equiv x^2 + y^2 = r^2$ b. Tangent perpendicular to the line OP c. Line gradient $OP = m_{OP} = \frac{y_1}{x_1}$ $l \perp OP$, then $m_l \cdot m_{OP} = -1$ $m_l \cdot \frac{y_1}{x_1} = -1$ $m_l = -\frac{x_1}{y_1}$ (line gradient)	3
		a. Equality n Circle $L \equiv x^2 + y^2 = r^2$ b. Tangent perpendicular to the line OP c. Line gradient $OP = m_{OP} = \frac{y_1}{x_1}$ $l \perp OP$, then $m_l \cdot m_{OP} = -1$ $m_l \cdot \frac{y_1}{x_1} = -1$ $m_l = -\frac{x_1}{y_1}$ (line gradient) d. By substituting the line gradient in the straight line equation, the equation of the tangent to the circle centered on the point will be obtained $O(0,0)$	4
6	 <p>Check whether the equation is tangent to the circle $x^2 + y^2 = 20$</p>	No answer	0
		Is known : Circle Equations: $x^2 + y^2 = 20$ Wanted: y? Answer:	1
		Is known :	2

	drawn from a point (10,0) outside the circle ie $2x + 4y = 20$	<p>Circle Equation: $x^2 + y^2 = 20$</p> <p>Asked: y?</p> <p>Answer:</p> <p>Circle equation (1)</p> <p>Polar equation for point (10,0)</p> $x_1x + y_1y = r^2$ $10x + 0y = 20$ $10x = 20$ $x = \frac{20}{10}$ $x = 2.... (2)$	
		<p>Is known :</p> <p>Circle Equations: $x^2 + y^2 = 20$</p> <p>Wanted: y?</p> <p>Answer:</p> <p>Circle equation $x^2 + y^2 = 20.... (1)$ The polar equation for the point (10,0)</p> $x_1x + y_1y = r^2$ $10x + 0y = 20$ $10x = 20$ $x = \frac{20}{10}$ $x = 2.... (2)$ <p>From equations 1 and 2</p> $x^2 + y^2 = 20$ $2^2 + y^2 = 20$ $4 + y^2 = 20$ $y^2 = 20 - 4$ $y^2 = 16$ $y = 4$ <p>Obtained tangent point (2,4) Equation of tangent at point (2,4)</p> $x_1x + y_1y = r^2$ $2x + 4y = 20$	3
		<p>Is known :</p> <p>Circle Equations: $x^2 + y^2 = 20$</p>	4

		<p>Wanted: y?</p> <p>Answer:</p> <p>Circle equation $x^2 + y^2 = 20$... (1) The polar equation for the point $(10,0)$</p> $x_1x + y_1y = r^2$ $10x + 0y = 20$ $10x = 20$ $x = \frac{20}{10}$ $x = 2$... (2) <p>From equations 1 and 2</p> $x^2 + y^2 = 20$ $2^2 + y^2 = 20$ $4 + y^2 = 20$ $y^2 = 20 - 4$ $y^2 = 16$ $y = 4$ <p>Obtained tangent point $(2,4)$ Equation of tangent at point $(2,4)$</p> $x_1x + y_1y = r^2$ $2x + 4y = 20$ <p>So, it is proved that the equation of the tangent is $2x + 4y = 20$</p>	
7	<p>Circle $x^2 + y^2 - 2x + 13y + 40 = 0$ intersects the Y axis at $y = -5$. Check what the line equals the point that is $3y - 2x + 15 = 0$!</p>	No answer	0
		<p>Is known:</p> $x^2 + y^2 - 2x + 13y + 40 = 0$ <p>intersects the Y axis at $y = -5$</p> <p>Asked: y?</p> <p>Answer:</p>	1
		<p>Is known:</p> $x^2 + y^2 - 2x + 13y + 40 = 0$ <p>intersects the Y axis at $y = -5$</p> <p>Asked: y?</p> <p>Answer:</p> <p>Hence the circle intersects the Y axis at $y = -5$ then the point of tangency to the circle is at coordinates $(0, -5)$.</p>	2
		<p>Is known:</p> $x^2 + y^2 - 2x + 13y + 40 = 0$	3

		<p>intersects the Y axis at $y = -5$ Asked: y? Answer:</p> <p>Hence the circle intersects the Y axis at $y = -5$ then the point of tangency to the circle is at coordinates (0, -5).</p> <p>The equation of the circle deduction line at point (0, -5):</p> $x_1x + y_1y + \frac{A}{2}(x + x_1) + \frac{B}{2}(y + y_1) + C = 0$ $\Leftrightarrow 0x + (-5)y - (x + 0) + \frac{13}{2}(y - 5) + 40 = 0$ $\Leftrightarrow -5y - x + \frac{13}{2}y - \frac{65}{2} + 40 = 0$ $\Leftrightarrow \frac{3}{2}y - x + \frac{15}{2} = 0$ $\Leftrightarrow 3y - 2x + 15 = 0$	
		<p>Is known: $x^2 + y^2 - 2x + 13y + 40 = 0$ intersects the Y axis at $y = -5$ Asked: y? Answer:</p> <p>Hence the circle intersects the Y axis at $y = -5$ then the point of tangency to the circle is at coordinates (0, -5).</p> <p>The equation of the circle deduction line at point (0, -5):</p> $x_1x + y_1y + \frac{A}{2}(x + x_1) + \frac{B}{2}(y + y_1) + C = 0$ $\Leftrightarrow 0x + (-5)y - (x + 0) + \frac{13}{2}(y - 5) + 40 = 0$ $\Leftrightarrow -5y - x + \frac{13}{2}y - \frac{65}{2} + 40 = 0$ $\Leftrightarrow \frac{3}{2}y - x + \frac{15}{2} = 0$ $\Leftrightarrow 3y - 2x + 15 = 0$ <p>So it is proven that the equation of the tangent is $3y - 2x + 15 = 0$</p>	4

Appendix 16

VALIDATION SHEET TEST OF UNDERSTANDING MATHEMATIC CONCEPTS

Education Unit : SMA Adabiah 2 Padang
Subjects : Mathematics
Class / Semester : XI / 2
Subject : Equations of Tangents to Circles

Put a check mark (✓) in the assessment column below:

No	Indicators Of The Ability To Understand Mathematical Concepts	No Question	Validity			Suggestion
			Valid	Little Improvement	Many Improvements	
1	Expressing the concepts that have been learned	1	✓			
2	Classify objects based on whether or not the requirements that form the concept are met	2	✓			
3	Identify the properties of the operation or concept	3		✓		
4	Apply concepts logically	4	✓			
5	Provide examples or examples of cons (not examples) of the concept being studied	5	✓			
6	Linking various concepts in mathematics and outside mathematics	6		✓		
7	Develop the necessary or sufficient conditions of a concept	7	✓			

Padang, February 2020

Dr. Yerizon, .M.Si

Appendix 17

FINAL ASSESSMENT OF CLASS XI STUDENTS SEMESTER OF SMA ADABIAH PADANG STUDY YEAR 2019/2020

No.	Class XI MIA 1
1	45
2	57
3	64
4	50
5	57
6	52
7	30
8	43
9	41
10	26
11	45
12	50
13	43
14	57
15	47
16	50
17	43
18	32
19	35
20	47
21	53
22	38
23	57
24	64
25	44
26	30
Average Flat	46.15

Appendix 18

**TEST VALUE DISTRIBUTION TRIAL THE TEST
OF UNDERSTANDING MATHEMATIC
CONCEPTS**

NO	Participant Name Code Educate	Score Item X							Total Score (X_t)
		1	2	3	4	5	6	7	
1	U1	1	3	2	1	0	1	1	9
2	U2	1	4	4	3	1	0	0	13
3	U3	2	4	4	4	2	2	0	18
4	U4	1	3	4	4	2	0	2	16
5	U5	1	4	4	3	1	0	0	13
6	U6	2	4	3	4	1	1	4	19
7	U7	1	4	3	3	1	0	0	12
8	U8	1	3	3	2	2	1	0	12
9	U9	1	4	4	4	2	0	0	15
10	U10	2	4	4	3	2	0	2	17
11	U11	1	4	4	3	1	0	0	13
12	U12	1	3	3	2	0	1	0	12
13	U13	1	4	4	4	1	2	1	17
14	U14	3	4	4	3	2	1	4	21
15	U15	3	4	4	3	1	2	1	18
16	U16	2	3	3	1	0	1	1	11
17	U17	2	3	4	3	1	0	0	13
18	U18	1	4	4	3	1	0	1	14
19	U19	1	3	3	3	0	1	0	11
20	U20	1	3	4	3	2	0	1	14
21	U21	2	4	4	4	2	0	2	18
22	U22	1	4	4	3	2	0	2	16
23	U23	1	4	4	3	2	0	2	16
24	U24	2	4	4	4	1	0	0	15

Appendix 19

PROBLEM ITEM DIFFERENT INDEX TABLE

<i>Df</i>	0.10	0.05	0.02	0.01
1	t = 6.34	t = 12.71	t = 31.82	t = 63.66
2	2.92	4.30	6.96	9.92
3	2.35	3.18	4.54	5.48
4	2.13	2.78	3.75	4.60
5	2.02	2.57	3.36	4.03
6	1.94	2.45	3.14	3.71
7	1.90	2.36	3.00	3.50
8	1.86	2.31	2.90	3.36
9	1.83	2.26	2.82	3.25
10	1.81	2.23	2.76	3.17
11	1.80	2.20	2.72	3.11
12	1.78	2.18	2.68	3.06
13	1.77	2.16	2.65	3.01
14	1.76	2.14	2.62	2.98
15	1.75	2.13	2.60	2.95
16	1.74	2.12	2.58	2.92
17	1.73	2.11	2.57	2.90
18	1.73	2.10	2.54	2.90
19	1.72	2.09	2.53	2.86
20	1.72	2.08	2.52	2.84
21	1.72	2.07	2.51	2.83
22	1.71	2.07	2.50	2.82
23	1.71	2.06	2.48	2.81
24	1.71	2.06	2.48	2.78
25	1.71	2.06	2.48	2.78
26	1.71	2.05	2.47	2.77
27	1.70	2.05	2.47	2.76
28	1.70	2.04	2.46	2.75
29	1.70	2.04	2.46	2.72
30	1.69	2.03	2.44	2.71
35	1.68	2.02	2.42	2.69
40	1.68	2.02	2.41	2.68
45	1.68	2.01	2.40	2.66
50	1.67	2.00	2.39	2.65
60	1.67	2.00	2.38	2.64
70	1.66	1.99	2.38	2.63
80	1.66	1.98	2.37	2.63
90	1.66	1.98	2.36	2.62
100	1.66	1.98	2.36	2.61
125	1.66	1.98	2.35	2.60
150	1.65	1.97	2.35	2.59
200	1.65	1.97	2.34	2.59
300	1.65	1.97	2.34	2.59
400	1.65	1.96	2.33	2.59
500	1.65	1.96	2.33	2.58
1000	1.65	1.96	2.33	2.58

Appendix 20

CALCULATION OF DIFFERENT INDEX FOR TRIAL PROBLEMS

Test participants (N) = 24 people

$$n = 27\% \times N = 27\% \times 24 = 6,48 \approx 6 \text{ learners}$$

$$n_t = n_r = n = 6$$

So, each of the 6 students for the high group and the low group.

$$d_f = (n_t - 1) + (n_r - 1) = (6 - 1) + (6 - 1) = 10$$

For an error rate of 5% at df = 10 we get $I_p \text{ table} = 2.23$

Problem Number 1

No.	High Group			Low Group		
	Name	Score	$(x_t - M_t)^2$	Name	Score	$(x_r - M_r)^2$
1	U14	3	0.705	U1	1	0.025
2	U6	2	0.025	U16	2	0.705
3	U3	2	0.025	U19	1	0.025
4	U15	3	0.705	U7	1	0.025
5	U21	2	0.025	U8	1	0.025
6	U13	1	1,345	U12	1	0.025
total		13	2.83	total	7	0.83
Mt		2.16		Mr	1.16	

$$I_p = \frac{Mt - Mr}{\sqrt{\frac{\sum X_t^2 + \sum X_r^2}{n(n-1)}}} = \frac{2,16 - 1,16}{\sqrt{\frac{2,83 + 0,83}{30}}} = \frac{1}{0,349} = 2,86$$

$I_p \text{ count} > I_p \text{ table}$, then question number 1 is categorized as significant.

Problem Number 2

No.	High Group			Low Group		
	Name	Score	$(X_t - M_t)_2$	Name	Score	$(X_r - M_r)_2$
1	U14	4	0	U1	3	0.025
2	U6	4	0	U16	3	0.025
3	U3	4	0	U19	3	0.025
4	U15	4	0	U7	4	0.705
5	U21	4	0	U8	3	0.025
6	U13	4	0	U12	3	0.025
total		24	0	total	19	0.83
Mt		4		Mr	3.16	

$$I_p = \frac{Mt - Mr}{\sqrt{\frac{\sum X_t^2 + \sum X_r^2}{n(n-1)}}} = \frac{4 - 3,16}{\sqrt{\frac{0 + 0,83}{30}}} = \frac{0,84}{0,16} = 5,25$$

I_p count > I_p table, then question number 2 is categorized as significant.

Problem Number 3

No.	High Group			Low Group		
	Name	Score	$(X_t - M_t)_2$	Name	Score	$(X_r - M_r)_2$
1	U14	4	0.028	U1	2	0.688
2	U6	3	0.688	U16	3	0.028
3	U3	4	0.028	U19	3	0.028
4	U15	4	0.028	U7	3	0.028
5	U21	4	0.028	U8	3	0.028
6	U13	4	0.028	U12	3	0.028
total		23	0.828	total	17	0.828
Mt		3.83		Mr	2.83	

$$I_p = \frac{Mt - Mr}{\sqrt{\frac{\sum X_t^2 + \sum X_r^2}{n(n-1)}}} = \frac{3,83 - 2,83}{\sqrt{\frac{0,828 + 0,828}{30}}} = \frac{1}{0,23} = 4,34$$

I_p count > I_p table, then question number 3 is categorized as significant.

Problem Number 4

No.	High Group			Low Group		
	Name	Score	$(x_t - M_t)_2$	Name	Score	$(x_r - M_r)_2$
1	U14	3	0.435	U1	1	1
2	U6	4	0.115	U16	1	1
3	U3	4	0.115	U19	3	1
4	U15	3	0.435	U7	3	1
5	U21	4	0.115	U8	2	0
6	U13	4	0.115	U12	2	0
total		22	1.33	total	12	4
Mt		3.66		Mr	2	

$$I_p = \frac{Mt - Mr}{\sqrt{\frac{\sum Xt^2 + \sum Xr^2}{n(n-1)}}} = \frac{3,66 - 2}{\sqrt{\frac{1,33 + 4}{30}}} = \frac{1,66}{0,42} = 3,95$$

I_p count > I_p table, then question number 4 is categorized as significant.

Problem Number 5

No.	High Group			Low Group		
	Name	Score	$(x_t - M_t)_2$	Name	Score	$(x_r - M_r)_2$
1	U14	2	0.25	U1	0	0.25
2	U6	1	0.25	U16	0	0.25
3	U3	2	0.25	U19	0	0.25
4	U15	1	0.25	U7	1	0.25
5	U21	2	0.25	U8	2	2.25
6	U13	1	0.25	U12	0	0.25
total		9	1.5	total	3	3.5
Mt		1.5		Mr	0.5	

$$I_p = \frac{Mt - Mr}{\sqrt{\frac{\sum Xt^2 + \sum Xr^2}{n(n-1)}}} = \frac{1,5 - 0,5}{\sqrt{\frac{1,5 + 3,5}{30}}} = \frac{1}{0,4} = 2,5$$

I_p count > I_p table, then question number 5 is categorized as significant.

Problem Number 6

No.	High Group	Low Group
-----	------------	-----------

	Name	Score	$(X_t - Mt)_2$	Name	Score	$(X_r - Mr)_2$
1	U14	1	0.108	U1	1	0.028
2	U6	1	0.108	U16	1	0.028
3	U3	2	0.448	U19	1	0.028
4	U15	2	0.448	U7	0	0.688
5	U21	0	1,768	U8	1	0.028
6	U13	2	0.448	U12	1	0.028
total		8	3,328	total	5	0.828
Mt		1.33		Mr	0.83	

$$I_p = \frac{Mt - Mr}{\sqrt{\frac{\sum Xt^2 + \sum Xr^2}{n(n-1)}}} = \frac{1,33 - 0,83}{\sqrt{\frac{3,328 + 0,828}{30}}} = \frac{0,5}{0,138} = 3,62$$

I_p count > I_p table, then question number 6 is categorized as significant.

Problem Number 7

No.	High Group			Low Group		
	Name	Score	$(X_t - Mt)_2$	Name	Score	$(X_r - Mr)_2$
1	U14	4	4	U1	1	0.448
2	U6	4	4	U16	1	0.448
3	U3	0	4	U19	0	0.108
4	U15	1	1	U7	0	0.108
5	U21	2	0	U8	0	0.108
6	U13	1	1	U12	0	0.108
total		12	14	total	2	1,328
Mt		2		Mr	0.33	

$$I_p = \frac{Mt - Mr}{\sqrt{\frac{\sum Xt^2 + \sum Xr^2}{n(n-1)}}} = \frac{2 - 0,33}{\sqrt{\frac{14 + 1,328}{30}}} = \frac{1,67}{0,714} = 2,33$$

I_p count > I_p table, then question number 7 is categorized as significant

Appendix 21

CALCULATION OF ADVANTAGES OF TRIAL PROBLEMS

Problem Number 1

No · ·	High Group		Low Group	
	Name	Score	Name	Score
1	U14	3	U1	1
2	U6	2	U16	2
3	U3	2	U19	1
4	U15	3	U7	1
5	U21	2	U8	1
6	U13	1	U12	1
Amount (D _t)		13	Amount (D _r)	7

$$I_k = \frac{D_t + D_r}{2mn} \times 100\%$$

$$= \frac{13 + 7}{2 \cdot 3 \cdot 6} \times 100\% = 55,55 \%$$

The difficulty index for question number 1 is moderate

Problem Number 2

No · ·	High Group		Low Group	
	Name	Score	Name	Score
1	U14	4	U1	3
2	U6	4	U16	3
3	U3	4	U19	3
4	U15	4	U7	4
5	U21	4	U8	3
6	U13	4	U12	3
Amount (D _t)		24	Amount (D _r)	19

$$I_k = \frac{D_t + D_r}{2mn} \times 100\%$$

$$= \frac{24 + 19}{2 \cdot 4 \cdot 6} \times 100\% = 89,58 \%$$

The difficulty index question number 2 is easy

Problem Number 3

No · ·	High Group		Low Group	
	Name	Score	Name	Score
1	U14	4	U1	2
2	U6	3	U16	3
3	U3	4	U19	3
4	U15	4	U7	3
5	U21	4	U8	3
6	U13	4	U12	3

$$I_k = \frac{D_t + D_r}{2mn} \times 100\%$$

$$= \frac{23 + 17}{2 \cdot 4 \cdot 6} \times 100\% = 83,33 \%$$

The difficulty index question number 3 is easy

Amount (D _t)	23	Amount (D _r)	17	
--------------------------	----	--------------------------	----	--

Problem Number 4

No	High Group		Low Group		
	Name	Score	Name	Score	
1	U14	3	U1	1	$I_k = \frac{D_t + D_r}{2mn} \times 100\%$ $= \frac{22 + 12}{2.4.6} \times 100\% = 70,83 \%$ <p>The difficulty index for question number 4 is moderate</p>
2	U6	4	U16	1	
3	U3	4	U19	3	
4	U15	3	U7	3	
5	U21	4	U8	2	
6	U13	4	U12	2	
Amount (D _t)		22	Amount (D _r)	12	

Problem Number 5

No.	High Group		Low Group		
	Name	Score	Name	Score	
1	U14	2	U1	0	$I_k = \frac{D_t + D_r}{2mn} \times 100\%$ $= \frac{9 + 3}{2.4.6} \times 100\% = 25 \%$ <p>The difficulty index question number 5 is difficult</p>
2	U6	1	U16	0	
3	U3	2	U19	0	
4	U15	1	U7	1	
5	U21	2	U8	2	
6	U13	1	U12	0	
Amount (D _t)		9	Amount (D _r)	3	

Problem Number 6

No.	High Group		Low Group		
	Name	Score	Name	Score	
1	U14	1	U1	1	$I_k = \frac{D_t + D_r}{2mn} \times 100\%$ $= \frac{8 + 5}{2.4.6} \times 100\% = 27,08 \%$ <p>The difficulty index for question number 6 is moderate</p>
2	U6	1	U16	1	
3	U3	2	U19	1	
4	U15	2	U7	0	
5	U21	0	U8	1	
6	U13	2	U12	1	
Amount (D _t)		8	Amount (D _r)	5	

Problem Number 7

No.	High Group	Low Group	
			$I_k = \frac{D_t + D_r}{2mn} \times 100\%$

.	Name	Score	Name	Score	$= \frac{12 + 2}{2.4.6} \times 100\% = 29,16\%$ <p>The difficulty index for question number 7 is moderate</p>
1	U14	4	U1	1	
2	U6	4	U16	1	
3	U3	0	U19	0	
4	U15	1	U7	0	
5	U21	2	U8	0	
6	U13	1	U12	0	
Amount (D _t)		12	Amount (D _r)	2	

Appendix 22

CLASSIFICATION OF TEST RESULTS

TRIAL TEST OF UNDERSTANDING

$I_p \text{ table} = 2.23$ MATHEMATIC CONCEPTS

The classification of questions used is as follows:

- Problem is used if I_p is significant and $0\% < I_k < 100\%$.
- Corrected test questions if:
 - I_p is significant and $I_k = 0\%$ or $I_k = 100\%$.
 - I_p is not significant and $0\% < I_k < 100\%$.
- Test questions are replaced if I_p is not significant and $I_k = 0\%$ or $I_k = 100\%$.

No. Question	Distinguishing Index		Difficulty index		Classification
	I_p	Question criteria	I_k	Question Criteria	
1	2.86	Significant	55.55%	Moderate	Put on
2	5.25	Significant	89.58%	Easy	Put on
3	4.34	Significant	83.33%	Easy	Put on
4	3.95	Significant	70.83%	Moderate	Put on
5	2.5	Significant	25%	Hard	Put on
6	3.62	Significant	27.08%	Moderate	Put on
7	2.33	Significant	29.16%	Moderate	Put on

Appendix 23

CALCULATION OF RELIABILITY TRIAL PROBLEMS OF UNDERSTANDING MATHEMATIC CONCEPT TEST

NO.	Student Name Code	Score Item X							Total Score (X_t)	$(X_t)^2$
		1	2	3	4	5	6	7		
1	U1	1	3	2	1	0	1	1	9	81
2	U2	1	4	4	3	1	0	0	13	169
3	U3	2	4	4	4	2	2	0	18	324
4	U4	1	3	4	4	2	0	2	16	256
5	U5	1	4	4	3	1	0	0	13	169
6	U6	2	4	3	4	1	1	4	19	361
7	U7	1	4	3	3	1	0	0	12	144
8	U8	1	3	3	2	2	1	0	12	144
9	U9	1	4	4	4	2	0	0	15	225
10	U10	2	4	4	3	2	0	2	17	289
11	U11	1	4	4	3	1	0	0	13	169
12	U12	1	3	3	2	0	1	0	12	144
13	U13	1	4	4	4	1	2	1	17	289
14	U14	3	4	4	3	2	1	4	21	441
15	U15	3	4	4	3	1	2	1	18	324
16	U16	2	3	3	1	0	1	1	11	121
17	U17	2	3	4	3	1	0	0	13	169
18	U18	1	4	4	3	1	0	1	14	196
19	U19	1	3	3	3	0	1	0	11	121
20	U20	1	3	4	3	2	0	1	14	196
21	U21	2	4	4	4	2	0	2	18	324
22	U22	1	4	4	3	2	0	2	16	256
23	U23	1	4	4	3	2	0	2	16	256
24	U24	2	4	4	4	1	0	0	15	225
$\sum x_i$		35	88	88	73	30	13	24	353	5393
$(\sum x_i)^2$		1225	7744	7744	5329	900	169	576	124609	
$\sum x_i^2$		61	328	330	239	50	19	58	5393	

Calculation of the variance of the score for each question: The variance of the score for question number 1

$$\begin{aligned}\sigma_1^2 &= \frac{\sum x_1^2 - \frac{(\sum x_1)^2}{N}}{N} \\ &= \frac{61 - \frac{1225}{24}}{24} \\ &= 0,41\end{aligned}$$

The variant score for question number 2

$$\begin{aligned}\sigma_2^2 &= \frac{\sum x_2^2 - \frac{(\sum x_2)^2}{N}}{N} \\ &= \frac{328 - \frac{7744}{24}}{24} \\ &= 0,22\end{aligned}$$

The variant score for question number 3

$$\begin{aligned}\sigma_3^2 &= \frac{\sum x_3^2 - \frac{(\sum x_3)^2}{N}}{N} \\ &= \frac{330 - \frac{7744}{24}}{24} \\ &= 0,30\end{aligned}$$

The variant score for question number 4

$$\begin{aligned}\sigma_4^2 &= \frac{\sum x_4^2 - \frac{(\sum x_4)^2}{N}}{N} \\ &= \frac{239 - \frac{5329}{24}}{24} = 0,70\end{aligned}$$

The variant score for question number 5

$$\begin{aligned}\sigma_5^2 &= \frac{\sum x_5^2 - \frac{(\sum x_5)^2}{N}}{N} \\ &= \frac{50 - \frac{900}{24}}{24} \\ &= 0,53\end{aligned}$$

The variant score for question number 6

$$\begin{aligned}\sigma_6^2 &= \frac{\sum x_6^2 - \frac{(\sum x_6)^2}{N}}{N} \\ &= \frac{19 - \frac{169}{24}}{24} \\ &= 0,49\end{aligned}$$

The variant score for question number 7

$$\begin{aligned}\sigma_7^2 &= \frac{\sum x_7^2 - \frac{(\sum x_7)^2}{N}}{N} \\ &= \frac{58 - \frac{576}{24}}{24} \\ &= 1,41\end{aligned}$$

Variance of total score

$$\begin{aligned}\sigma_t^2 &= \frac{\sum x_t^2 - \frac{(\sum x_t)^2}{N}}{N} \\ &= \frac{5393 - \frac{124609}{24}}{24} \\ &= 8,37\end{aligned}$$

The reliability of the test test questions

$$\begin{aligned}r_{11} &= \left(\frac{n}{n-1}\right) \left(1 - \frac{\sum \sigma_i^2}{\sigma_t^2}\right) \\ &= \left(\frac{7}{7-1}\right) \left(1 - \frac{4,06}{8,37}\right) \\ &= \left(\frac{7}{6}\right) (0,51493) \\ &= 0,60075\end{aligned}$$

Obtained reliability test test questions of 0,60075 means test questions has high reliability.

Appendix 25

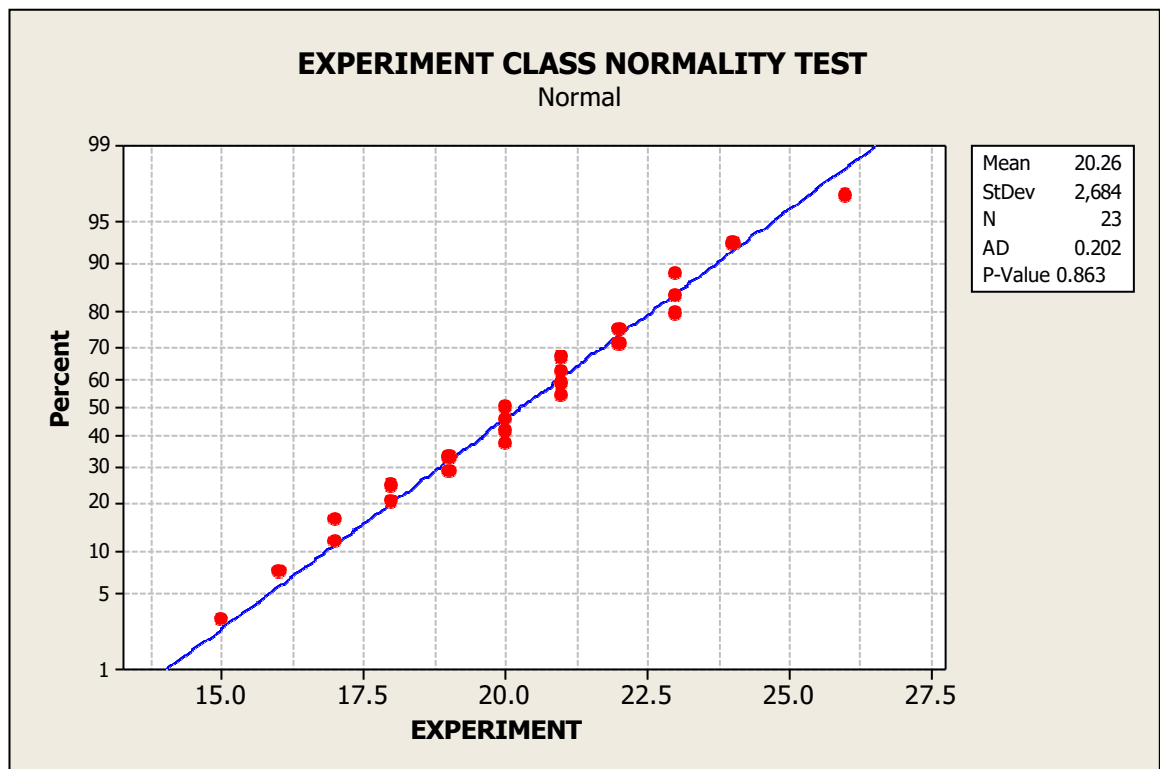
DISTRIBUTION OF CONTROL CLASS TEST SCORES

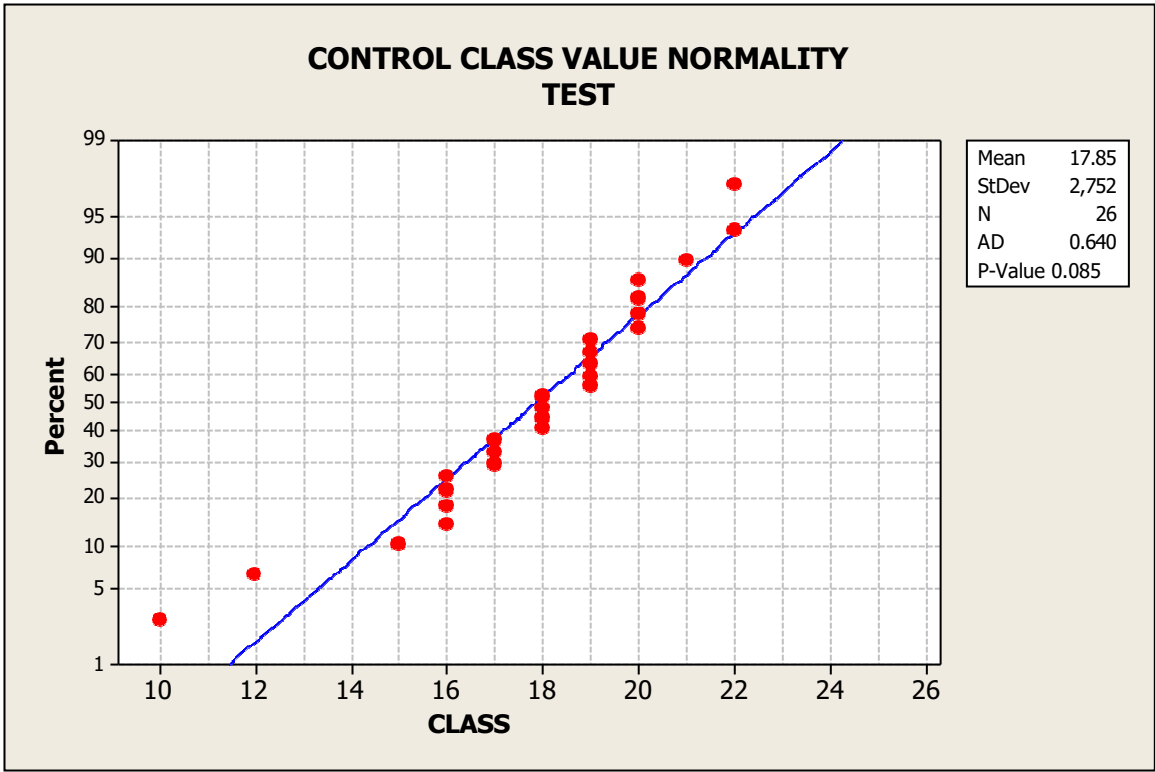
No.	Name	Score about item X							Total Score
		1	2	3	4	5	6	7	
1	AW	3	3	3	3	2	3	3	20
2	DF	3	4	0	2	3	0	0	12
3	FAF	2	2	3	3	2	3	3	18
4	FAN	2	4	2	3	3	2	3	19
5	FAI	3	4	3	1	2	3	0	16
6	GR	1	3	3	3	3	3	2	18
7	K	3	4	3	3	2	2	3	20
8	MI	2	2	3	2	2	3	2	16
9	MZ	3	4	0	3	3	3	3	19
10	MN	1	4	2	4	3	3	0	17
11	NH	3	3	2	3	3	3	3	20
12	RWZ	3	2	3	3	1	3	3	18
13	RA	3	4	3	1	3	2	0	16
14	RRH	2	4	4	3	3	3	3	22
15	R	3	4	3	3	3	2	0	18
16	RDJ	2	4	3	3	3	3	2	20
17	REN	3	4	2	4	2	2	0	17
18	SF	3	4	3	1	3	2	3	19
19	SN	1	2	3	3	2	3	3	17
20	SS	3	2	3	2	2	3	0	15
21	VS	3	4	2	4	2	3	3	21
22	TMA	1	2	0	2	2	0	3	10
23	KAJ	3	4	3	4	3	2	3	22
24	HM	1	4	2	1	2	3	3	16
25	N	1	3	3	3	3	3	3	19
26	LE	3	4	2	3	2	2	3	19
total		61	88	63	70	64	64	54	
Average		2.34	3.38	2.42	2.69	2.46	2.46	2.07	
TOTAL									464
AVERAGE									17.85
MAXIMUM VALUE									22

MINIMUM VALUE	10
DEVIATION STANDARD	2.75

Appendix 26

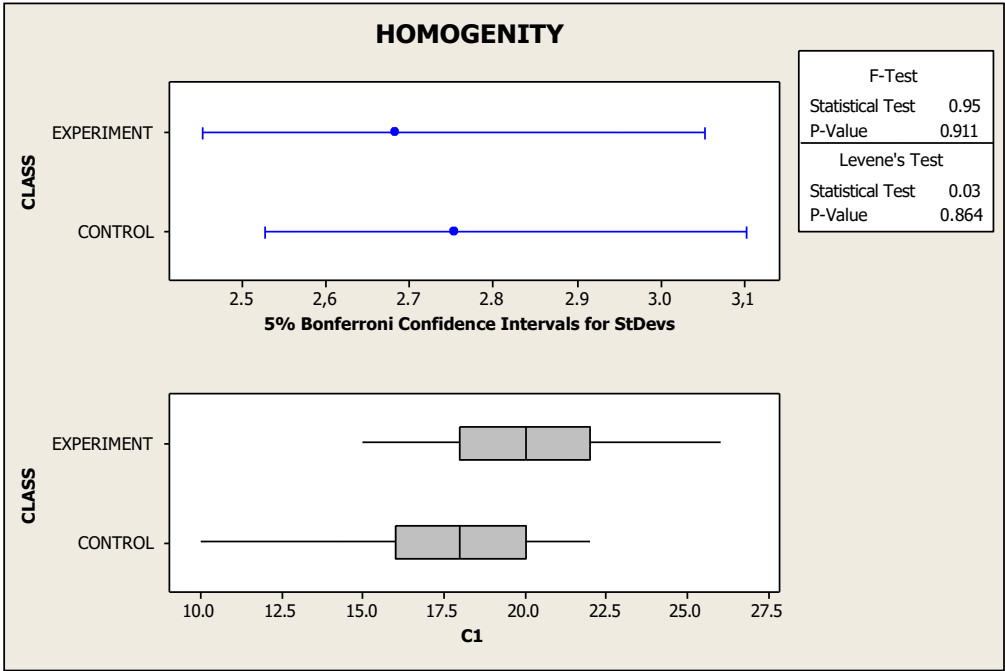
SAMPLE CLASS NORMALITY TEST





Appendix 27

HOMOGENITY TEST OF SAMPLE CLASS VARIANCE



Appendix 28

RESEARCH HYPOTHESIS TEST

wo-Sample T-Test and CI: EXPERIMENT CLASS; CLASS CONTROL

Two-sample T for experimental class vs

	control	class	C2	N	Mean	StDev	SE
EXPERIMENT	23	20.26	2.68		0.56		
CONTROL	26	17.85	2.75		0.54		

Difference = mu (EXPERIMENT) - mu
(CONTROL) Estimate for difference:
2,415

95% CI for difference: (0.850; 3,980)

T-Test of difference = 0 (vs not =): T-Value = 3.11 P-Value = 0.003 DF
= 48

Appendix 29


KEMENTERIAN PENDIDIKAN DAN KEBUDAYAAN
UNIVERSITAS NEGERI PADANG
FAKULTAS MATEMATIKA DAN ILMU PENGETAHUAN ALAM
 Alamat : Jln. Prof. Dr. Harkus Air Tawar Padang 25131 Telepon : 0751-7057420
 Website : <http://www.unp.ac.id>

Nomor : SP1/UNP/1099/2020 03 Februari 2020
 Hal : Surat Perintah

Yth. Kepala Dinas Pendidikan Provinsi Sumatera Barat
 Jl. Jendral Sudirman No.32A Jati Baru
 di
 Padang

Dengan hormat

Bersama ini kami sampaikan bahwa dalam rangka menyelesaikan Tugas Akhir/Skripsi, mahasiswa kami dari jenjang pendidikan S1 PMIPA UNP bermaksud akan melaksanakan Penelitian di SMA Adabiah 2 Padang.

Sebagaimana dengan hal di atas, kami mohon persetujuan Bapak/Ibu dan mohonkan surat izin melakukan Penelitian untuk mahasiswa tersebut di bawah ini:

No	Nama / NIM / Pendidikan	Tempat Penelitian	Waktu Penelitian	Judul Skripsi / Tugas Akhir
1	Vanya Ardenthy / 100250440 / Pendidikan Matematika / S1	SMA Adabiah 2 Padang	17-02-2020 03 15-03-2020	Pengantar Penerapan Model Pembelajaran Kooperatif Tipe Think Talk Write Terhadap Penalaran Konsep Matematika Variasi Teks Terna TS QMA, SMA Adabiah 2 Padang

Demikian surat pengantar ini kami sampaikan. Sehubungan dengan hal tersebut kami mohon kerukunannya.


 Yohanes M. S. P. D.
 NIM 100217221600041003

Lampiran:
 1. Deklarasi PMIPA UNP, sebagai laporan
 2. Surat Izin dari Dinas Pendidikan PMIPA UNP
 3. Kopra SMA Adabiah 2 Padang



Appendix 30

PEMERINTAH PROVINSI SUMATERA BARAT
DINAS PENDIDIKAN
Jln. Jenderal Sudirman No. 52 Tel. p. (2781) 22162-21956 Fax (2781) 37047 - 37010 Padang

Nomor : 420.02/ HK /PSMA 2020
Lampiran : -
Perihal : Izin Uji Coba Soal

Padang 11 Februari 2020

Kepada Yth :
Wakil Dekan I FMIPA
Universitas Negeri Padang
di
Padang

With this letter number 802 / U
N35.1 / PP / 2020 dated Febru-
ary 4, 2020 regarding the test o-
f questions for undergraduate
students of the Mathematics E-
ducation Study Program, with t-
he title "The Effect of the Applica-
tion of Cooperative Learning
Model Types of Critical Think T-
alk on Understanding the Math-
ematical Concepts of Class Stu-
dents" di SMA Adabiah 2 Pada-
ng Universitas Negeri Padang,
on behalf of:

Nama : Vanya Aridanthry
NIM : 16029040
Tempat : SMA Adabiah Padang
Waktu : 17 Februari 2020 s.d 15 Maret 2020

In this regard, the West Sumatra
a Provincial Education Office in
principle does not mind giving
permission to students whose
names are above to carry out t-
est questions, but it is hoped th-
at during the activity it can pay
attention to the following matte-
rs:

1. Coordinate with the principal
mentioned above
2. Does not interfere with the te-
aching and learning process ac-
tivities in the education unit
3. Does not burden the financin-
g of students in the education
unit
4. Test questions are carried o-
ut entirely for educational purp-
oses and not to be published in
general
5. The data is taken in accorda-
nce with the applicable laws an-
d regulations
6. After completing the test que-
stions, please submit a report t-
o the West Sumatra Provincial
Education Office.

Thus we provide this letter, so t-
hat it can be used properly.

a.n. Kepala
Sekretaris,

Drs. Syofrizal, B.MT
NIP. 19620405 198903 1 008

Tembusan disampaikan kepada, rub:

1. Gubernur Sumatera Barat (sebagai laporan)
2. Kepala sekolah yang terkait

Appendix 31

PEMERINTAH PROVINSI SUMATERA BARAT
DINAS PENDIDIKAN
Jln. Jenderal Sudirman No. 22, Tel. p. (0781) 20100 - 21888 Fax. (0781) 37547 - 37510 Padang

Nomor : 420.02/ HK /PSMA 2020
Lampiran :
Perihal : Izin Uji Coba Soal

Padang, 11 Februari 2020

Kepada Yth. :
Wakil Dekan I FMIPA
Universitas Negeri Padang
di
Padang

With this letter number 802 / U
K35.1 / PP / 2020 dated Febru-
ary 4, 2020 regarding the test o-
f questions for undergraduate
students of the Mathematics E-
ducation Study Program, with t-
he title "The Effect of the Applica-
tion of Cooperative Learning
Model Types of Critical Think-
g on Understanding the Math-
ematical Concepts of Class Stu-
dents XI SMA Adabiah 2 Padan-
g Universitas Negeri Padang,
on behalf of:

Nama : Vanya Aridanthry
NIM : 18029040
Tempat : SMA Adabiah Padang
Waktu : 17 Februari 2020 s.d 15 Maret 2020

In this regard, the West Sumatra Provincial Education Office in principle does not mind giving permission to students whose names are above to carry out test questions, but it is hoped that during the activity it can pay attention to the following matters:

1. Coordinate with the principal mentioned above
2. Do not interfere with the teaching and learning process activities in the education unit
3. Does not burden the financing of students in the education unit
4. Test questions are carried out entirely for educational purposes and not to be published in general
5. The data is taken in accordance with the applicable laws and regulations
6. After completing the test questions, please submit a report to the West Sumatra Provincial Education Office.

Thus we provide this letter, so that it can be used properly.


a.n. Kepala
Sekretaris,

Drs. Syofrizal, B.MT
NIP. 19620405 198903 1 008

Tembusan disampaikan kepada, yaitu:

1. Gubernur Sumatera Barat (sebagai laporan)
2. Kepala sekolah yang terkait

Aappendix 32


KEMENTERIAN PENDIDIKAN DAN KEBUDAYAAN
UNIVERSITAS NEGERI PADANG
FAKULTAS MATEMATIKA DAN ILMU PENGETAHUAN ALAM
 Alamat : Jln. Prof. Dr. Hamka Air Tawar Padang 25131 Telepon : 0751-7053420
 Website : <http://fmipa.unp.ac.id>


Nomor : 802 /UN35.1/PP/2020
 Hal : Isi Uji Coba Soal
04 Februari 2020

Yth. Kepala Dinas Pendidikan Provinsi Sumatera Barat
 Jl. Jendral Sudirman No.52A Jati Baru
 di
 Padang

With those latter, we give permission to do question test to the researcher;


No	Nama / NIM / Prodi	Tempat Penelitian	Waktu Penelitian	Judul Skripsi / Tugas Akhir
1	Vanya Aridanthi / 16029040 / Pendidikan Matematika / S1	SMA Adabiah 2 Padang	17-02-2020 s/d 15-03- 2020	EFFECT OF APPLICATION OF COOPERATIVE LEARNING MODEL OF THINK TALK WRITE TYPE ON UNDERSTANDING THE MATHEMATICAL CONCEPT OF STUDENTS IN GRADE XI SMA ADABIAH 2 PADANG

Demikianlah kami sampaikan, atas bantuan dan kerjasama Saudara kami ucapkan terima kasih.


 Yohandri, M. Sc. Ph.D.
 NIP. 197307252006041003

Tembusan:

1. Dekan FMIPA UNP, sebagai laporan.
2. Ketua Jurusan Matematika FMIPA UNP,
3. Kepala SMA Adabiah Padang



Appendix 33

	YAYASAN SYARIKAT OESAHA ADABIAH PADANG SEKOLAH MENENGAH ATAS (SMA) ADABIAH 2 AKREDITASI "A"
Jl. Jati Adabiah No 1 Padang Telp. (0751) 28765 Email : smaadabiah2padang@yahoo.co.id Web. http://www.smaadabiah2-pdg-sch.id	
Certificate	
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Thus we give this letter so that it is used as needed.	
Padang, 21 April 2020	
  Dra. Ratna Gustiherlina, M Pd NIP. 19641113 199103 2 001	