MATHEMATICAL MODEL OF ZAKAT FOR POVERTY REDUCTION

UNDERGRADUATE THESIS

As a partial fulfillment for bachelor degree in science

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بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

"Sesungguhnya bersama kesukaran itu ada keringanan. Karena itu bila kau sudah selesai (mengerjakan yang lain). Dan berharaplah kepada Tuhanmu"

(Q.S: Al-Insyirah: 6-8)

"Niscaya Allah akan mengangkat (derajat) orang-orang yang beriman diantaramu dan orang-orang yang diberi ilmu beberapa derajat"

(Q.S: Al-Mujadilah: 11)

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satu cita telah tercapai

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Engkau berikan kesempatan sampai pada saat awal perjuanganku.

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Bukan pelangi namanya jika hanya ada warna merah
Bukan hari namanya jika hanya ada siang
Semua itu adalah warna hidup yang harus dijalani
Meski terasa berat,
Manisnya hidup akan terasa, apabila semua bisa dilalui dengan baik,
Teruslah berusaha, belajar, dan berdo'a
Untuk menggapainya
Never Give Up!

Hanya sebuah karya kecil dan untaian kata-kata ini yang dapat

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"Tekatkan Niat, Berusaha Keras dan

Bersungguh dalam Do'a"-By: Lani Widia Putri

ABSTRACT

Lani Widia Putri: Zakat Mathematical Model for Poverty Reduction

Poverty is a condition that is interpreted as a lack of income to meet basic needs in life. Poverty has many negative impacts on survival and solutions must be found. Islam recognizes a term known as "zakat" as a mechanism to help the poor. Optimizing the collection of zakat funds can be done by turning people who have not paid zakat into zakat payers, making zakat funds more productive for the recipient. The wealth displacement from on to another community in economy using zakat can be represented by a mathematical model in which variables and parameters are controllable so that zakat is potentially effective for reducing poverty

This research is a basic research using descriptive method specifically by analyzing the theories relevant to the problem. This research begins at establishing a mathematical model of zakat for poverty reduction, in which zakat true debtors (i.e those obliged and pay zakat) ($\bar{K}$), discouraged zakat debtors (obliged but do not pay zakat) ($\bar{T}$), and zakat recipents ($\bar{Q}$), interact each other by certain factors. A fixed point would be obtained after the model contruscted followed by performing an analysis to test stability of the fixed point.

The results show that in the model of poverty reduction using zakat a fixed point, $E = (K^*, T^*)$ is obtained with $Q^* = 1 - K^* - T^*$, which is tested stable. Then by using several parameters values it is found that the factors affecting optimal poverty reduction are the zakat effect on economy, economic growth and the rate of interaction (between true and discouraged debtors). The rate of interaction tends to be stable faster than other factors, therefore the higher the rate of interaction the better and more optimal zakat reduces the poverty.

Keywords: *Mathematical Model, Zakat, Poverty.*

FOREWORD

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TABLE OF CONTENTS

ABSTRACT .... **i**
FOREWORD .... **ii**
TABLE OF CONTENTS .... **iv**
LIST OF TABLES .... **vi**
LIST OF FIGURES .... **vii**
LIST OF APPENDICES .... **viii**
CHAPTER I INTRODUCTION **1**
A. Research Background 1
B. Problem Formulation 5
C. Approach and Research Questions 5
D. Research Objectives 5
E. Research Benefits 6
F. Research Procedures 6
CHAPTER II THEORITICAL REVIEW **8**
A. Poverty 8
1. Definition of poverty 8
2. Causes of poverty 8
3. Criterions and forms of poverty 9
B. Zakat 10
1. Definition of Zakat 10
2. Type of assets paid for zakat 11
3. Conditions for paying zakat 12
4. Payment of Zakat 13
5. Recipients of Zakat 14
6. Obligation Criteria of Zakat 15
7. Punishment for Denying Zakat Obligation 16
C. Zakat and Poverty 17

D. Mathematical Modeling 19
1. Mathematical Model 19
2. Deterministic model 22
E. Differential Equations 23
F. Matrix 26
1. Eigen values 26
2. Linearance and Jacobian Matrix 27
G. Stability theory 28
1. Fixed point 29
2. Fixed point stability analysis 29
CHAPTER III RESULTS AND DISCUSSION 31
A. Mathematical model of zakat for poverty reduction 31
B. Analysis Results 40
1. Fixed Point of the Model 40
2. Fixed point stability test in various cases 44
3. Fixed point stability simulation 47
C. Model Interpretation 50
CHAPTER IV SUMMARY 54
A. Conclusion 54
B. Recommendations 56
BIBLIOGRAPHY 58
APPENDIX 60

LIST OF TABLES

Table 2.1. Amount of Zakat Collected in Padang City 2008-2015.................. 18

Table 2.2. Percentage of Poor Population in Padang City, 2008-2015............. 19

LIST OF FIGURES

Figure 2.1. Steps of Constructing a Mathematical Model.................................. 21

Figure 2.2. Infection calculation in an interaction communication $\boldsymbol{i, j = 1, \dots, m}$.. 23

Figure 3.1 Mathematical model chart of Zakat in Poverty Reduction.............. 33

Figure 3.2 Trajectory around the fixed point for Case 9.................................. 48

Figure 3.3 Trajectory around the fixed point for Case 12................................. 49

LIST OF APPENDICES

Appendix 1. MAPLE™ syntax in the simulation to find the fixed point for Case 9 61

Appendix 2. MAPLE™ syntax in the simulation to find the fixed point for Case 12 62

Appendix 3. Proof for eigen values.. 63

CHAPTER I
INTRODUCTION

A. Research Background

Islam commands its followers to maintain the relationship with Allah and human with aim at gaining happiness and prosperity in living today and in the hereafter. In simple terms, the relationship with Allah means that a Muslim must be sincere in all his activities only for worshipping Allah. Meanwhile, relationship with human means that a Muslim must have concern for others. Concern for others is an obligation that Muslim has a responsibility to provide solutions to the social problems, including poverty.

Poverty has become a global problem encountered by many countries nowadays, including Indonesia. It means that poverty is a crucial problem that needs to be overcome especially by policy makers and leaders of the country. This is in line with Kusuma and Sukmana's statement (2010: 3) that Indonesia is recognized as the largest Muslim population (approximately 129 million people) who are trapped in poverty with an income of less than US$ 2 per day or equivalent to Rp. 26,000.

Poverty occurs due to several factors such as limited natural resources, low quality of human resources and the lack of access to capital which result in underdevelopment of running businesses and low levels of production in goods and services.

The first effect of poverty is unemployment, which makes people hard or unable to fulfill their daily needs and further, decreases people's competitiveness and purchasing power. Second, unemployment may promote crimes or acts of violence. Third, poverty causes low education and leads to low competitiveness in international labour market especially in this era of globalization. Fourth, it leads to low health levels. There are still many negative consequences arising from poverty problems.

Islam recognizes a mechanism called zakat which has principally become an important cornerstone for Islamic socio-economic framework. Zakat is one of the five pillars a Muslim is obligated to pay with certain condition. The command on paying Zakat is strictly after the commad to establish shalat, meaning the importance of Zakat is vital for Muslim. The commandment on paying zakat has been delivered since the foundation of Islam, brought by the prophet and passed on by the companions.

The word zakat is mentioned in Qur'an about 82 times where 27 times of them is juxtaposed with word shalat (prayers). Al-Qur'an explains that people who are sincere in paying zakat are highly praised and those who deliberately leave the obligation will be punished. In the Al-Qur'an Fussilat verse 6-7 Allah SWT says:

قُلْ اِنَّمَآ اَنَا۠ بَشَرٌ مِّثْلُكُمْ يُوْحٰٓى اِلَيَّ اَنَّمَآ اِلٰهُكُمْ اِلٰهٌ وَّاحِدٌ فَاسْتَقِيْمُوْٓا اِلَيْهِ
وَاسْتَغْفِرُوْهُ ۗ وَوَيْلٌ لِّلْمُشْرِكِيْنَ ۙ (6)

الَّذِيْنَ لَا يُؤْتُوْنَ الزَّكٰوةَ وَهُمْ بِالْاٰخِرَةِ هُمْ كٰفِرُوْنَ (7)

Translation:

"*Say, (O Prophet), "I am only a man like you, (but) it has been revealed to me that your God is only One God. So take the Straight Way towards Him, and seek His forgiveness. And woe to the polytheists (6), those who do not pay zakat and are in denial of the Hereafter (7)"*.

The issue of zakat is an urgent concern and it is necessary to carry out socialization and in-depth studies about zakat, as well as optimizing the role of amil zakat (zakat distributors) institutions and increasing the role of the government in its management. If zakat processed more effectively and efficiently, zakat certainly has enormous benefits, one of which is that it can fight poverty and socio-economic problems that occur in society.

Zakat has actually been proven effectively reduced the poverty in Selengor, Malaysia as found by Patmawati (2006) in Beik (2009: 4). Patmawati is a zakat expert from Universiti Malaya (UM) who found that zakat can reduce the number of poor families and the level of the depth and severity of poverty in the Selangor state, Malaysia. The income gap can be reduced by 10 percent where 10 percent of society can enjoy 10 percent of wealth due to zakat. If without zakat, then the bottom group of people will only enjoy 9.6 percent of the wealth. While the top 10 percent enjoy 32 percent of wealth, or down from 35.97 percent without zakat. This proves that with zakat the gap between groups can be reduced.

Data obtained from the National Zakat Agency (BAZNAS) of Padang City and Statistics (BPS) of Padang Municipality in 2008-2015 shows that the amount of zakat collected shows a trend of increase by years as well as a trend of decrease in the number and percentage of poor people. Particularly, in 2010 and 2015 there was an increase in the percentage of poor people for several reasons, but in general, the increase in the amount of zakat collected has been associated with the decrease in the percentage of poor people.

Rachim (2012: 2) states that according to Dr. Yusuf Qardhawi, one of the fiqh scholars stated that one of the fundamental efforts to alleviate or reduce the problem of poverty is by optimizing the implementation of zakat. That is because zakat is a source of funds that will never run dry and run out. In other words, as long as Muslims have the awareness to give zakat and as long as the zakat funds are managed properly, zakat funds will always exist and are beneficial for the interests and welfare of the community.

In this research, a mathematical model is built by formulating real problems into mathematical form by taking into account the factors that are considered relevant in the problem. The mathematical model built will be analyzed in order to test it representativeness on the problems discussed. Based on the mathematical model, the relationship between these factors will be revealed. By knowing the relationship between these factors, it is expected to monitor changes in community's economy with the existence of zakat.

Since poverty reduction is an urgent topic, this research is making effort at providing a way to see changes in the level of the community's economy as well as improving the economy of the poor when zakat is implemented. Based on the above background, this research will construct a mathematical model of zakat so that this research is entitled "**Mathematical Model of Zakat for Poverty Reduction**".

B. Problem Formulation

Based on the background of the problems previously described, the formulation of the problems discussed in this study is how the mathematical model of zakat in poverty reduction?

C. Approach and Research Questions

Based on the research background and problem stated above, the research approach used literature study on relevant books and journals as well as data and material obtained from the internet on the issues discussed.

The research questions that will be answered in this study are:

1. What is the mathematical model of zakat in poverty reduction?
2. What are the results of the analysis of the mathematical model of zakat in poverty reduction?
3. What interpretation can be obtained from the analysis of the mathematical model of zakat in poverty reduction?

D. Research Objectives

Based on the problems raised above, the objectives of this study are:

1. Establishing a mathematical model of zakat in poverty reduction.
2. Analyzing the mathematical model of zakat in poverty reduction.

3. Interpreting the results of the analysis of the mathematical model of zakat in poverty reduction.

E. Research Benefits

The benefits of this research are:

1. This research provides an overview for researchers and readers to find out how the mathematical model of zakat in poverty reduction
2. The results of the analysis provided are expected to provide an overview of the zakat payer/debtor (*Muzzaki*) and zakat recipient (*Mustahiq*) so that they can become the basis for making the right policies for poverty reduction.
3. As input for further researchers in developing and expanding the scope of research.

F. Research Procedures

This research is basic research (theoretical) that implement an analysis using relevant theories on the problems discussed. The following steps taken in this research:

1. Identifying the real problem to be discussed i.e the issue of zakat which aims at reducing poverty.
2. Collecting theories relevant to the problem of zakat.
3. Determining the method to construct the mathematical model of zakat implementation on reducing poverty.

4. In constructing the model, first step is to determine the variables, parameters, and assumptions related to the problem of zakat in poverty reduction.
5. Analyzing the mathematical model obtained by finding the model's equilibrium point and testing the stability of the model's equilibrium point.
6. Interpreting the analysis results of the model obtained.
7. If the results of the analysis are not satisfying, the model is reconstructed while checking the assumptions.
8. Drawing conclusions from the analysis results.

CHAPTER II
THEORITICAL REVIEW

A. Poverty

1. Definition of poverty

Poverty is a condition that is interpreted as a lack of income to meet basic necessities of life. In other terms, poverty is an inability to meet basic needs that causes experience anxiety, misery or poverty in every step of life (Ahmadi, 2003: 344).

According to the Indonesian Formal Dictionary, poverty is a condition in which there is a shortage of ordinary things to have such as food, clothing, shelter and drinking water while these things strongly affect to the quality of life. Poverty sometimes also means that there is no access to education and jobs that are principally the solution for poverty problems.

2. Causes of poverty

Ahmadi (2003: 344) states that, there are several factors that cause poverty, as follows:

a. Low level of education

   Low educated people are associated with lacking of life skill that support work productivity. It is obviously an obstacle to find a job in the market.

b. Limited natural resources

   Poverty will hit a society if its natural resources no longer provide benefits for their life. It is often said by experts that people are poor because they are basically "poor by nature". Naturally poor means lack of endowment provided by the nature for example rocky lands which make impossible to cultivate and grow plants, lack of essential minerals and so on. Therefore, poor nature leads to poor people.

c. Limited job opportunity

Limited employment opportunities lead to poverty in community because it will be diffciult to open new jobs due to lack of skills and capital

d. Limited capital

Limited capital is a common phenomenon in developing countries. People become poor because they are lacking of capital to equip tools or materials in order to apply the skill they have to earn an income.

e. Family burdens

More family members bring more burden to reach prosperity.

3. Criterions and forms of poverty

According to Qadir (2001: 50), there are 3 criteria that belong to the poor. The criteria are as follows:

a. Those who do not have ability to carry out regular work at all due to several factors of age (elderly) or because of physical disabilities
b. Those who are capable of working physically but do not have any life skill.
c. Those getting poor for typical reason caused by a disaster, while physically and mentally they still have the potential to work, in addition to having no capital.

According to Baswir (1997: 23) and Sumodiningrat (1998: 90). Socio-economically, there are two forms of poverty, namely:

a. Absolute poverty is a poverty type determined by an income level below the poverty line or the amount of income is not sufficient to meet the minimum needs of life. Minimum living needs are measured by fulfillment for the need for food, clothing, health, housing and education, calories, GNP per capita, consumption expenditure and others. The World Bank defines absolute poverty as living with an income below USD 1/day and medium poverty with an income below USD 2.

b. Relative poverty is poverty that rises from comparing an income level with another income level. For example, someone who is classified as rich (capable) in rural area is not necessarily rich in urban areas, or even being poor.

This study refers to the absolute poverty concept. Those are classified as poor using the absolute poverty concept shall receive a number of assistance in the form of zakat distributed and hence the poor is called as zakat recipent (mustahiq).

B. Zakat

1. Definition of Zakat

Zakat is the payment of certain amount of a Muslim's wealth with some conditions must be met. The collected zakat shall be given to the group of people who deserve it and determined by the law of Islam. Mukri (2011: 2-11) explains zakat as follows:

Zakat literally means as growing and purifying. Meanwhile according to the law of Islam it means as an obligation of paying an amount of assets given to certain groups, or more precisely the eight groups mentioned in the chapter At-Taubah in Quran.

Considering that one's assets consist of several types, therefore not all types of assets owned must be paid for zakat. The type of assets that will be issued for zakat determined by certain criteria mentioned in Holy Qur'an and Hadith of the Prophet Muhammad.

2. Type of assets paid for zakat

According to the Indonesian Ministry of Religion (2013: 8-10), zakat is basically divided into two types, namely zakat mal and zakat fitrah. Zakat fitrah is zakat that must be issued by every individual Muslim during the holy month of Ramadhan. Meanwhile, zakat mal is zakat that associated to many types of assets.

According to Hasan (2008: 24), the Al-Qur'an explains that only a few types of assets that must be calculated in order to issue the zakat part, such as:

a. Gold and silver, as mentioned in the in Surah At-Taubah verse 34.
b. Professional salaries, as explained in the Surah Az-Zariyat verse 19 and Al-Baqarah verse 267
c. Agricultural crops and fruit crops, as stated in the Qur'an, surah Al-An'am verse 141.
d. Livestocks, as mentioned in Surah An-Nahl verses 5-7. Types of livestock that are associated with zakat are cow and buffalo, goat and sheep, horses and certain other livestocks
e. Trade assets

   Trade assets mean traded assets, as stated in surah Al-Baqarah verse 267.

f. Mining assets

As described in surah Al-Baqarah verse 267.

g. General wealth assets

As stated in Quran surah At-Taubah verse 103.

Apart from the types of assets that will be issued for zakat, zakat cannot be implemented carelessly. A person who wants to pay zakat must meet certain conditions in order to pay zakat.

3. Conditions for paying zakat

According to the Indonesian Ministry of Religion (2013: 25), the requirements for people who are obliged to zakat are as follows:

a. Being mu'min and muslim

Zakat is one of the pillars of Islam, therefore it is obligatory on its (mu'min) believers and muslim only. The difference between a believer and a muslim lies on the determination, sincerity and fear when practicing the religious teachings. The mu'mins only believe in Islam teachings but do not necessarily practice the teaching as done by the muslims.

b. Being sane

Islam does not oblige zakat for people who does not have sane senses. The obligation for zakat on assets is borne by the guardian or the person who manages the property, such as an orphan who has assets and has met the requirements for zakat.

c. The assets have reached the nishab

d. Not under slavery.

e. The issued assets for zakat have been obligated to pay for zakat
f. The asset status is a full property athat can be fully utilized and owned in a way that is justified by thesyariah or obtained in a lawful way.
g. The assets have been owned for at least a year following the Qamariah year system
h. The assets are not debt
i. The assets have passed over the fulfillment of basic needs.

After the conditions of zakat have been fulfilled, someone is obliged to pay zakat, but not everyone who has met such requirements is willing to pay zakat. Given that everyone has their own opinions, habits, ways and lifestyle, so that the zakat payers can be classified into several groups.

4. Payment of Zakat

According to Al-Shaykh (2008: 32-33), people can be divided into the following when it comes at their response on paying the zakat:

a. Zakat payers are those who recognize zakat as an obligation and pay it at a predetermined time and amount.
b. Those who postpone the payment of zakat without renouncing the obligation.
c. Those who refusal to pay zakat and deny this obligation and therefore are considered as non-believers whose status is equal to idol worshippers.

Those who do not pay zakat is caused by several things including a lack of understanding of zakat, lack of motivation to pay zakat and many other factors. This can be overcome by inviting and motivating groups that have not paid zakat to pay zakat. In accordance with Harafah (2010: 4) statements that zakat payment can be made by inviting people who have excess assets to pay zakat in order to purify their assets. This is expected to encourage this group to become zakat payers.

Furthermore, in this study zakat payments are divided into 2 groups: those obliged to pay zakat then paying the zakat (*muzzaki*) and those obliged to pay zakat but not paying zakat. After someone pay the zakat to a zakat management institution, the zakat will be distributed to those who are entitled to receive zakat or known as *mustahiq*. The criterions for some people entitled to receive zakat have been explained in the Qur'an.

5. Recipients of Zakat

According to Al-Shaykh (2008: 86-90), there are 8 groups according to Al-Qur'an in surah At-Taubah verse 60 who are entitled to receive zakat, as follows:

a. Al-Fuqara '(extremely poor people), a person who do not have any assets or have some assets but less than half of the basic needs to be fulfilled.
b. Al-Masakin (poor people), is a person who has assets half of life needs or more but is not sufficient to fulfill the needs.
c. Al-Amilin'Alaiha (zakat collectors), are people who are appointed by the leader of the Islamic state or the governor in the activity of collecting zakat.
d. Mu'Alaf Qulubihim (people with weak heart), is a person who is just converted to Islam and need to be strengthened.
e. Fi Riqab (slave), is a person who wants to free himself from slavery.
f. Al-Gharimin, is a person who is burdened with debt and cannot pay this debt.

g. Fi Sabilillah (people who are led by Allah), is a person who has spent his entire life fighting in the way of Allah SWT.

h. Ibn Sabil (Wanderer), are the people traveling or traveler and do not have money to return to their home land.

The recipients of zakat ashould be also bounded by age. Tjiptoherijanto (2001: 4), states that in population structure, the productive age group ranges from 15-64 years old. Therefore, it is more beneficial and effective that in population the age of zakat payers and zakat recipients also ranges from 15 to 64 years since the range can guarantee better productivity of zakat.

Initially, this study classifies the community into two economic groups, namely the economically advantaged people as zakat payers and the economically disadvantaged people as zakat recipients. However, the reality tells that not all of the economically advantaged people pay zakat. Therefore the community is divided into 3 groups.

6. Obligation Criteria of Zakat

According to the Ministry of Religion of the Republic of Indonesia (2013: 66), associated to professional zakat, it is a zakat that can be spent continuously every month if the income itself has passed its nishab as formulated below:

Professional zakat = 2.5% × (total income – routine payment or installment).

The nishab for professional zakat is 520 × the market price of one kilogram of rice.

Example:

Mrs. A has a salary of Rp. 2 million/month with additional income from the phone balance top-up of 8 million/month. Mrs. A pays a house installment of 5 million/month because Mrs. A does not yet have a house. The price of 1 kg of rice is around Rp. 8,000.00 per Kg.

Nishab of professional zakat = 520 × 8,000 = 4,160,000

Professional zakat = 2.5% × (10,000,000-5,000,000) = 125,000

because Mrs. A's net income has reached her nishab, she must pay Rp. 125.000 as zakat every month.

The illustration above has made clear that the criteria for zakat payers are groups that have a net income whose amount has reached the nishab. The same idea and implementation hold for agricultural assets, etc., which meet the predetermined conditions of zakat payment. With this mechanism, zakat shall open up the opportunity to reduce poverty.

7. Punishment for Denying Zakat Obligation

Mastur and Apat (2002: 10) state that the penalties for not paying zakat are as follows:

a. Burning in Saqar or Hell
b. Affected by slander/calamity or a painful punishment
c. Tormented with the assets possessed
d. Tortured with a snake bite
e. Tortured with hot stones
f. Unpeaceful heart

g. Immediate punishment

C. Zakat and Poverty

Zakat and poverty are closely related since the impact of zakat payment on economy is enormous. According to the Indonesian Ministry of Religion (2013), conceptually zakat has dimensions of poverty alleviation and reduction as well as reaching better social welfare. There are several strong reasons to believe that zakat as an instrument and pillar of Islam to prosper the community.

1. The use and allocation of zakat has been regulated by syariah where the recipients (*mustahiq*) only consist of eight groups (*asnaf*) by giving higher priority to the poor. This means that other groups outside the eight groups are not entitled to zakat, so the effectiveness of zakat at reducing poverty is more guaranteed.
2. Collection of zakat are sourced from various sources and community economic activities as explained in the type of assets that must be paid for zakat. Even in the present context, zakat can be taken from financial assets or certain professions. In short, zakat funding is potentially large.
3. Zakat is a teaching that is fulfilled by every Muslim who has excess assets continuously so that it will be a guarantee the receiving of zakat in a stable manner if it is managed properly. Thus, the stable condition of receiving zakat will also ensure the durability and sustainability of poverty alleviation programs. In another sense, the source of financing for poverty alleviation

programs from zakat will never run out because ideally Muslims will always pay zakat.

The management of zakat is currently aided by both government and private institutions. The following is data regarding the amount of zakat collected by one of the zakat management institutions, namely the National Zakat Agency (BAZNAS), in Padang City is shown in Table 2.1 as follows:

Table 2.1. Amount of Zakat Collected in Padang City 2008-2015

No	Tahun	Jumlah Zakat yang Terkumpul (milyar)
1	2008	2,4
2	2009	10,7
3	2010	11,5
4	2011	15,1
5	2012	19,7
6	2013	19,4
7	2014	20
8	2015	22,53

Sumber: Badan Amil Zakat Nasional (BAZNAS) of Padang Municipality

Based on the data above, it can be seen that the amount of zakat collected by BAZNAS Padang municipality has increased from year to year. So that the increase in the amount of zakat collected is expected to be able to help groups of people with weak economies who become the recipients of zakat.

As a proof that zakat has been well managed and and optimally utilized is the reduction trend in the number of weak economic groups classified as zakat recipients. The following will show the percentage of poor people in Padang municipality based on data obtained from Statistics (BPS) of Padang municipality in Table 2.2 as follows:

Table 2.2. Percentage of Poor Population in Padang City, 2008-2015

No	Tahun	Persentase Penduduk Miskin
1	2008	6.40
2	2009	5.72
3	2010	6.31
4	2011	6.02
5	2012	5.30
6	2013	5.02
7	2014	4.56
8	2015	4.93

Sumber: Badan Pusat Statistik Kota Padang

Optimal collection of zakat funds seems to potentially improve the community's economy, as an example shown in the Table 2.2 that the percentage of poor population follows a decreasing trend within eight years. It is fascinating to reveal the factors that affect such economic changes, as well as the relationship between these factors. To do so, a mathematical method is needed called mathematical modeling.

D. Mathematical Modeling

1. Mathematical Model

Mathematical models play an important role in science. Mathematical models are simple representations of real problems into mathematical expressions. Widowati and Sutimin (2007: 1) state that mathematical modeling is to represent and explain real-world problems in mathematical statements, so that an understanding of real-world problems becomes more precise. The mathematical representation of a phenomenon that results from the process of describing it in mathematical symbols is called a "Mathematical Model".

The mathematical model is a tool that helps solve real-world problems using mathematics. This mathematical model can be used in many fields of science, such as physics, biology, chemistry, economics, engineering, medicine, and so on.

In building a model, several steps are needed to produce a reliable model. According to Pagalay (2009: 5), the stages in building a model are as follows:

a. Identifying the problem. Problem identification is built from various questions that are raised concerning the problem. Weak problem identification often leads to an invalid model.
b. Building assumptions. Models are complex simplifications of reality. For this reason, every simplification requires assumptions, so that the scope of the model is within the problem corridor of relevancy.
c. Constructing the model. After building assumptions, the model is constructed using functional relationships by making diagrams, flowcharts, and mathematical equations.
d. Analyzing the model, means to determine the appropriate analysis and look for suitable solutions to answer the questions that are built in the problem identification stage.
e. Interpreting the model. The interpretation of the model is translating mathematical solutions into practical answers. This interpretation is important to determine whether the results obtained are reasonable or not.

f. Model validation. Model validation is carried out to verify the validity of the model designed with the assumptions previously built. A valid model means that it does not only following valid theoretical rules but also provide an interpretation of the results that are close to conformity. If most of these verification standards can be passed, the model can be implemented. Conversely, if not the model construction must be redesigned.

The following chart describes the steps taken to produce a mathematical model:

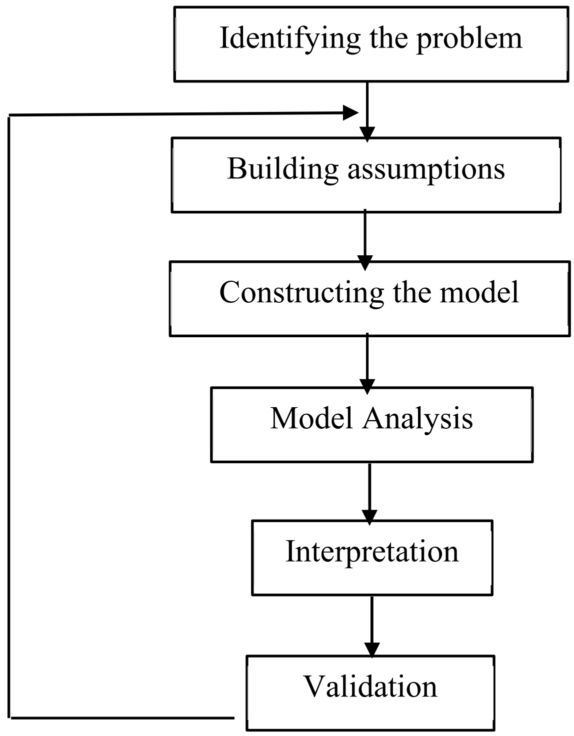

Figure 2.1. Steps of Constructing a Mathematical Model

By following the above processes, a mathematical model that represents how zakat works in reducing poverty can be constructed efficiently. The mathematical model of zakat distribution simulates some outspreads like those found in contagious diseases. This is often called as epidemic model. There are two types of mathematical models, namely deterministic models and stochastic models. The deterministic model, also known as the compartment model, individuals are categorized into different subgroups (compartments).

In this study, researchers divided individuals into several subgroups. Since zakat is a real-world problem and countable in definite value, then the deterministic model is considered more appropriate to use in this study.

2. Deterministic model

The deterministic model is a mathematical model in which any occurring symptoms can be measured with a fairly high degree of certainty and the events that occur have a probability that still does not contain a probabilistic (random) component. According to Widowati and Sutimin (2007: 2), deterministic models include the use of equations or a set of equations to present various components of a system or problem. In a deterministic model of neglected random variation, this equation is used to express a real-world problem which is formulated based on the basic relationships of the factors involved in this problem.

In the case of zakat, the interactions that occur between individuals who are obliged to zakat and pay zakat will be taken into account with individuals who are obliged to zakat but do not pay zakat, which affects each other. This is analogue to disease spread in a population that occurs rapidly which is called as epidemic.

According to Delay and Gani (2007: 21-23), the development of an epidemic process is deterministic in the sense that there is no random process in it. A simple epidemic in group interaction is for example the population consists of m groups of size $N_1, \dots, N_m$ with each other can change from one group to another and vice versa. Assume that there is an interaction between one group and another. The placement of a β infection measure, supposing that the group j is vulnerable of infection from group i for each pair β_{ij} calculation; for $i = j$ put $\beta_j = \beta_{jj}$, $(j = 1, \dots, m)$. Figure 2.2 illustrates the model from Rushton and Mautner (1995)

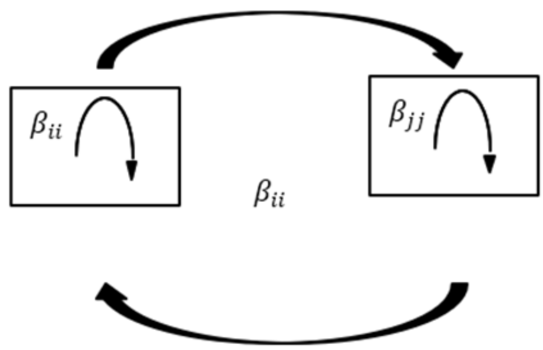

Figure 2.2. Infection calculation in an interaction communication $i, j = 1, \dots, m$

E. Differential Equations

The differential equation (DE) is an equation that contains the derivative of one dependent variable with respect to the other independent variables. DE is classified into two equations namely the ordinary differential equations (ODE) and partial differential equations (PDE).

Definition 1:

Ordinary differential equation is a differential equation that contains the function of one variable and its ordinary derivative of the function (Simon and Krantz, 2007: 4)

Example:

$$\frac{dy}{dx} = x + 10$$

$$\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2y = 0$$

Definition 2:

Partial differential equation is a differential equation that contains the function of two or more variables and their partial derivatives of these functions (Simon dan Krantz, 2007: 4)

Example:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + 8z = 0$$

A total of n interrelated differential equations will form a system called a system of differential equations which is defined as follows:

Definition 3:

A total of n interrelated differential equations will form a system called a system of differential equations.

Systems of differential equations can also be grouped based on the form of their equations, namely systems of linear differential equations and systems of nonlinear differential equations. The definition will be explained as follows:

The following system of first-order linear differential equations is given:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{A}\boldsymbol{x} + b(t)$$

where A is the coefficient matrix of size $n \times n$ and $b(t)$ is a continuous function. This system is called a first order linear differential equation system.

If $b(t) = 0$ then the system is said to be homogeneous and if $b(t) \neq 0$ then the system is said to be non-homogeneous. (Perko, 2001: 60)

Example:

$$\frac{dx_1}{dt} = 7x_1 - x_2 + 6x_3$$

$$\frac{dx_2}{dt} = -10x_1 + 4x_2 - 12x_3$$

$$\frac{dx_3}{dt} = -2x_1 + x_2 - x_3$$

The example above is a system of first-order homogeneous linear differential equations. The system of linear differential equations has been described above while the system of non-linear differential equations is as defined as follows:

Definition 4:

The system of first order nonlinear differential equations is stated as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\,(t, x)$$

with $\boldsymbol{x} = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$, and $\boldsymbol{f}(t, x) = \begin{pmatrix} f_1(t, x_1, \cdots, x_n) \\ f_2(t, x_1, \cdots, x_n) \\ \vdots \\ f_n(t, x_1, \cdots, x_n) \end{pmatrix}$

If $\boldsymbol{f}(t, x)$ is a non-linear function of $x_1, \cdots, x_n$ then the system is called as nonlinear differential equation (Perko, 2001: 65)

Example:

$$\frac{dx}{dt} = -2x + 3x^2$$

$$\frac{dy}{dt} = -3x + y^2 + 2x$$

The example above is a system of first order nonlinear differential equations because it contains the form of multiplication between the dependent variables, namely $3x^2$ and y^2.

F. Matrix

1. Eigen values

If A is a $n \; x \; n$ matrix, then the non zero vector X in $\mathbb{R}^n$ is called as eigen vector of A if AX is scalar multiplication of X,

$$AX = \lambda X$$

for any scalar λ. Scalar λ is called the eigen value of A and X is called the eigen vector related to λ. To calculate the eigen matrix A of $n \times n$, rewrite $AX = \lambda X$ as

$$AX = \lambda IX$$

or equivalently

$$(\lambda I - A)X = 0$$

In order for λ to be an eigen value, there must be a non-zero solution of the equation $(\lambda I - A)X = 0$. The equation $(\lambda I - A)X = 0$ will have a non-zero solution if and only if $det\ (\lambda I - A) = 0$.

The equation $det\ (\lambda I - A) = 0$ is called the characteristic equation A, the scalar that satisfies this equation is the eigen value of A. When extended, then $det\ (\lambda I - A) = 0$ is the λ polynomial which is called the characteristic of A (Anton dan Rorres, 2004: 384-385).

Example:

Find the eigen value of the following matrix:

$$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$$

Answer:

Eigen value is the solution of the equation det $(A - \lambda I) = 0$

$$\det\ (A - \lambda I) = \begin{vmatrix} 4-\lambda & 1 \\ 3 & 2-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(2-\lambda) - 3 = 0$$

$$\lambda^2 - 6\lambda + 8 = 0$$

The eigen values of A are $\lambda_1 = -2$ and $\lambda_2 = -4$

2. Linearance and Jacobian Matrix

System of nonlinear differential equations $\dot{x} = f(t, x)$ where $f(t, x)$ is a nonlinear function on $x_1, x_2, \dots, x_n$. This system of equations can be written explicitly as:

$$\begin{aligned} x_1' &= f_1(t, x_1, x_2, \cdots, x_n) \\ x_2' &= f_2(t, x_1, x_2, \cdots, x_n) \\ &\vdots \\ x_n' &= f_n(t, x_1, x_2, \cdots, x_n) \end{aligned}$$

while in matrix form the equation can be written as follows:

$$\begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{pmatrix} = \begin{pmatrix} f_1(t, x_1, x_2, \cdots, x_n) \\ f_2(t, x_1, x_2, \cdots, x_n) \\ \vdots \\ f_n(t, x_1, x_2, \cdots, x_n) \end{pmatrix}$$

The Jacobian matrix of the system of equation above (sometimes called "Jacobi" only) is defined as follows:

Definition 6 (Jacobian Matrix)

$$J(f(x)) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}$$

To determine the eigen value of the Jacobian matrix, one must solve the equation $det(\lambda I - J) = 0$ (Cain dan Reynold, 2010: 83)

G. Stability theory

In finding solutions to mathematical model problems related to the spread of disease, a theory of stability is needed. The stability of a system is overviewed at its fixed point.

1. Fixed point

To determine the fixed point of a system of differential equations the following definition is used:

Definition7 (Fixed point)

Suppose a system of differential equations is given as follows:

$$x' = f(x), x \in \mathbb{R}^n$$

Fixed point of $x' = f(x)$ is $\bar{x}$ such that $f(\bar{x}) = 0$. (Cain dan Reynold, 2010: 94)

Example:

The point (2,1,4) is the fixed point of the system below:

$$x_1' = 3x_1 + 2x_2 - 2x_3$$

$$x_2' = -x_1 - 2x_2 + x_3$$

$$x_3' = 2x_1 - 8x_2 + x_3$$

2. Fixed point stability analysis

The stability of the fixed point of the system can be determined based on the eigen values of the Jacobian matrix. In general, the stability of a fixed point according to Johnson (2006: 451) has the following criteria:

a. If all the eigen values are negative real, then the fixed point is stable.
b. If all the eigenvalues are positive real, then the fixed point is unstable.
c. If all the eigenvalues have different signs, then the fixed point is unstable.
d. If all eigenvalues are complex with a negative real part, then the fixed point is asymptotically stable.

e. If all the eigenvalues are complex with a positive real part, then the fixed point is unstable.

f. If all the eigenvalues are complex with zero real part, then the fixed point is stable but not asymptotically stable.

CHAPTER III
RESULTS AND DISCUSSION

A. Mathematical model of zakat for poverty reduction

The formation of a mathematical model of zakat for poverty reduction consider three groups of population as follows:

1. Groups of people who are obliged to do zakat and pay zakat (muzzaki)
2. Groups of people who are obliged to do zakat but do not pay zakat
3. Groups of people who are recipients of zakat (mustahiq).

Several factors affecting zakat payment in reducing poverty will be reviewed in the process of model formation. It is concerned that the number of individuals who are obliged to do zakat but do not pay zakat increases and this results in less zakat expected to escalate the economy of zakat recipients (mustahiq).

The following variables are used in the formation of the model of zakat for poverty reduction:

N : population

$\bar{K}$: the number of individuals obliged and pay zakat (muzzaki)

$\bar{T}$: the number of individuals obliged but do not pay zakat

$\bar{Q}$: the number of zakat receiving individuals (mustahiq)

Therefore, the population can be written: $N = \bar{K} + \bar{T} + \bar{Q}$. The parameters used in constructing the model are as follows:

j : interaction rate

μ : natural birth and mortality rate

r : failure rate in running the current business

p : the success rate in running a current business

z : zakat effect on economy

The next step is to construct the basic framework of the model by determining the assumptions to be used in the goal model. The assumptions used in this model are as follows:

1. Individuals obliged to zakat but do not pay zakat can become individuals who are obliged to zakat and pay zakat due to interactions (the interaction here is specifically in the form of invitations and motivation to pay zakat in order to open up the mind to be willing to pay the zakat).
2. This model only takes into account those obliged to pay zakat and those deserve to receive zakat.
3. The distribution of zakat received by zakat recipients (mustahiq) can help improve their economy.
4. The population is constant.
5. Constant rate of interaction.

Under these assumptions, variables and parameters can be charted in the following diagram:

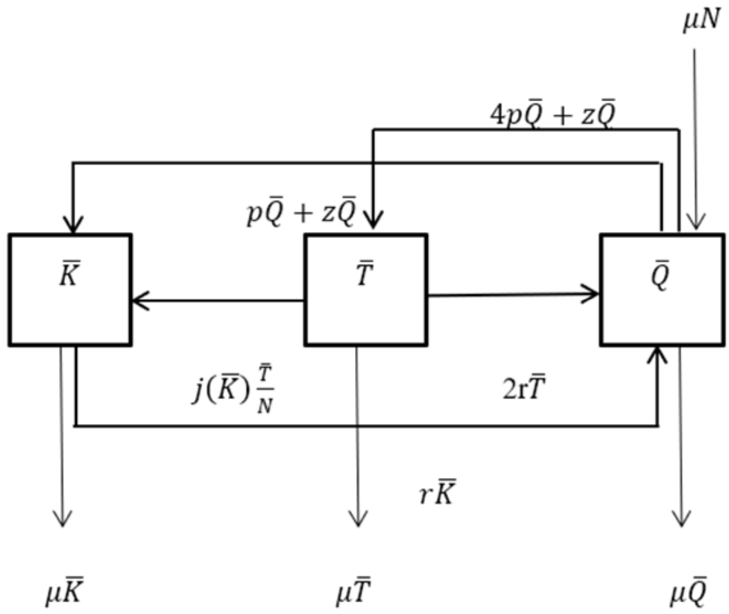

Figure 3.1 Mathematical model chart of Zakat in Poverty Reduction

Figure 3.1 may be interpreted as follows:

a. The rate of change in the number of individuals obliged to zakat and pay zakat (zakat debtors) is influenced by:

   1) The interaction rate that occurs between individuals who are obliged to pay zakat and pay zakat (muzzaki) and individuals who are obliged to do zakat but do not pay zakat potentially cause the latter becomes zakat payer, which is as large as:

$$j\bar{K}\frac{\bar{T}}{N} = j\left(\frac{\bar{K}}{\bar{\bar{N}}}\right)\bar{T}$$

   2) Optimistic economic growth may promote the zakat recipients to become zakat debtors since good growth may increase purchasing power and demand for business being run. If this affect the jobs or businesses being run by zakat recipients positevely (mustahiq) in the

sense that they accumulate more wealth over the nishab (which is now obliged to pay zakat), then the economic growth occurs as large as:

$$p\bar{Q} + z\bar{Q}$$

3) The rate of failure in running a business which results in individuals being obliged to zakat (muzzaki) and pay zakat to become zakat recipients (mustahiq), is calculated as large as:

$$-r\bar{K}$$

4) The number of natural deaths that occur in the group of individuals obliged to pay zakat and pay zakat, is calculated as large as:

$$-\mu\bar{K}$$

Based on the description above, the number of individuals who are obliged to do zakat and pay zakat will increase with an optimistic economic growth and the zakat distributed from these individuals can assist the business and economy of zakat recipients (mustahiq). It will also increase the interaction with individuals who are obliged to zakat but do not pay zakat thus that they become zakat payers. On the other side, the failure to run a business may cause individuals become poorer and no longer obliged to zakat, and furthermore may cause them become the recipients of zakat (mustahiq). Natural death also reduces the number of zakat debtors (muzakki). Therefore, the relationship between all variables and parameters in affecting the number of zakat debtors is expressed as follows:

$$\frac{d\bar{K}}{dt} = j\left(\frac{\bar{K}}{N}\right)\bar{T} + p\bar{Q} + z\bar{Q} - r\bar{K} - \mu\bar{K}$$

b. The rate of change in the number of individuals obliged to zakat but not pay zakat (discouraged zakat debtors) is influenced by:

1) The interaction that occurs between zakat debtors and discouraged zakat debtors may promote motivation to pay zakat and the number of those discouraged may decrease as large as:

$$-j\frac{\bar{T}}{N}\bar{K} = -j\left(\frac{\bar{K}}{N}\right)\bar{T}$$

2) Optimistic economic growth may promote the zakat recipients to become zakat debtors since good growth may increase purchasing power and demand for business being run. If this affect the jobs or businesses being run by zakat recipients positevely (mustahiq) in the sense that they accumulate more wealth over the nishab (which is now obliged to pay zakat). Mulkhan (2015: 2) estimated that there were 10-20% of people previously classified poor now become those obliged to pay zakat for having improvement in their economy. The economic growth is as large as:

$$4p\bar{Q} + z\bar{Q}$$

3) The rate of failure in running a business that results in discouraged zakat debtors become zakat recipients (mustahiq). According to Mastur and Apat (2002: 10) this is one of the punishments will be hastened in the world for those unwilling to pay zakat when they are obliged. Therefore, the rate of failure is calculated twice that of zakat debtors, as large as:

$$-2r\bar{T}$$

4) The number of natural deaths that occur in groups of discouraged zakat debtors is as large as:

$$-\mu\bar{T}$$

Based on the description above, the number of individuals of discouraged zakat debtors will increase with the economic growth but it will also be reduced by the interaction with zakat debtors (muzzaki) so that they become zakat payers. Severely, the failure to run a business makes the discouraged zakat debtors become the recipient of zakat (mustahiq). Naturally, the number of discouraged zakat debtors will also decrease with the occurrence of death. This relationship is stated as follows:

$$\frac{d\bar{T}}{dt} = 4p\bar{Q} + z\bar{Q} - j\left(\frac{\bar{K}}{N}\right)\bar{T} - 2r\bar{T} - \mu\bar{T}$$

c. The rate of change in the number of individual zakat recipients (mustahiq) is affected by:

1) Zakat may improve the business being run by zakat recipients (mustahiq) and promotes zakat recipients (mustahiq) to become zakat debtors. The economic growth or business success occuring regardless of zakat, in form of high purchasing power results in an increase in the number of zakat debtors. Mulkhan (2015: 2) states that it is estimated that people previously poor now are able to pay 10-20% zakat. The decrease in the number of zakat recipients (mustahiq) is as large as:

$$-(4p\bar{Q} + z\bar{Q})$$

2) The rate of failure in running a business that results in discouraged zakat debtors become zakat recipients (mustahiq). According to Mastur and

Apat (2002: 10) this is one of the punishments will be hastened in the world for those unwilling to pay zakat when they are obliged. Therefore, the rate of failure is calculated twice that of zakat debtors, as large as:

$$2r\bar{T}$$

3) The optimistic economic growth along with optimal zakat distribution shall escalate the business being run by zakat recipients (mustahiq) in a way so that now they become zakat debtors. The reduction of zakat recipents are calculated as large as:

$$-(p\bar{Q} + z\bar{Q})$$

4) The failure in running a business results in zakat debtors (muzzaki) to become zakat recipients (mustahiq), the increase in the number of zakat recipients (mustahiq) is as large as:

$$r\bar{K}$$

5) The number of births in the group of zakat debtors, the groups of discouraged zakat debtors and and the groups of zakat recipients is as large as:

$$\mu N$$

6) The number of natural deaths occuring in groups of zakat recipients (mustahiq) is as large as:

$$-\mu\bar{Q}$$

Based on the description above, the change in the number of zakat recipients (mustahiq) is reducible by the economic growth and optimal distribution of zakat since zakat help promoting the business of zakat recipients (mustahiq).

It is also reduced by natural death. At contrary, the failure to run business causes zakat debtors as well as discouraged zakat debtors is no longer obliged to pay zakat. Naturally, births increase the number of individuals in the three groups. It is written as the following equation:

$$\frac{d\bar{Q}}{dt} = 2r\bar{T} + r\bar{K} - p\bar{Q} - z\bar{Q} - 4p\bar{Q} - z\bar{Q} - \mu\bar{Q} + \mu N$$

Based on the above model formation process, a mathematical model of zakat for poverty reduction is obtained as follows:

$$\frac{d\bar{K}}{dt} = j\left(\frac{\bar{K}}{N}\right)\bar{T} + p\bar{Q} + z\bar{Q} - r\bar{K} - \mu\bar{K}$$

$$\frac{d\bar{T}}{dt} = 4p\bar{Q} + z\bar{Q} - j\left(\frac{\bar{K}}{N}\right)\bar{T} - 2r\bar{T} - \mu\bar{T}$$

$$\frac{d\bar{Q}}{dt} = 2r\bar{T} + r\bar{K} - 5p\bar{Q} - 2z\bar{Q} - \mu\bar{Q} + \mu N$$

where $N = \bar{K} + \bar{T} + \bar{Q}$.

To simplify the model analysis, normalization is used by defining new variables as follows:

$K = \frac{\bar{K}}{N}$ then $\bar{K} = KN$

$T = \frac{\bar{T}}{N}$ then $\bar{T} = TN$

$Q = \frac{\bar{Q}}{N}$ then $\bar{Q} = QN$

Subtituting the new variable definition above into all equations yields:

$$\frac{dKN}{dt} = j\left(\frac{KN}{N}\right)TN + pQN + zQN - rKN - \mu KN$$

$$\frac{dTN}{dt} = 4pQN + zQN - j\left(\frac{KN}{N}\right)TN - 2rTN - \mu TN$$

$$\frac{dQN}{dt} = 2rTN + rKN - 5pQN - 2zQN - \mu Q + \mu N$$

Multiply the above equations by $\frac{1}{N}$, a more simple and usable mathematical model is obtained, written as follows:

$$\frac{dK}{dt} = jKT + pQ + zQ - rK - \mu K \quad (3.1)$$

$$\frac{dT}{dt} = 4pQ + zQ - 2rT - jKT - \mu T \quad (3.2)$$

$$\frac{dQ}{dt} = 2rT + rK - 5pQ - 2zQ - \mu Q + \mu \quad (3.3)$$

Since $Q = \frac{\bar{Q}}{N} = \frac{N-\bar{K}-\bar{T}}{N} = \frac{N-KN-TN}{N} = 1 - K - T$ it is obtained $Q = 1 - K - T$, Since the population is constant, then in the analysis Q will be considered as the remaining of other groups. Substitute the equation $Q = 1 - K - T$ into equations (3.1) and (3.2) obtained

$$\frac{dK}{dt} = jKT + p(1 - K - T) + z(1 - K - T) - rK - \mu K$$

$$\frac{dT}{dt} = 4p(1 - K - T) + z(1 - K - T) - 2rT - jKT - \mu T$$

Thus,

$$\frac{dK}{dt} = jKT + pQ + zQ - rK - \mu K \quad (3.4)$$

$$\frac{dK}{dt} = jKT + p - pK - pT + z - zK - zT - rK - \mu K \quad (3.5)$$

$$\frac{dT}{dt} = 4p - 4pK - 4pT + z - zK - zT - 2rT - jKT - \mu T \quad (3.6)$$

The mathematical model of zakat for poverty reduction is in the form of a system of differential equations, so to validate the model, the limit values of 0 and 1 are used.

1. Validasi pada batas K

   If $K=0$, then $f'(0)=\frac{dK(0)}{dt}=(p+z)(1-T)$ thus $f'(0)$ is positive, therefore when $K=0$, the function $f=K$ is increasing. If $K=1$, then $f'(1)=\frac{dK(1)}{dt}=-r-\mu$ thus $f'(1)$ is negative. So when $K=1$, the function $f=K$ is decreasing.

2. Validasi pada batas T

   If $T=0$, then $f'(0)=\frac{dT(0)}{dt}=(2p+z)(1-K)$ thus $f'(0)$ is positive. It means that if $T=0$, then $f=T$ is increasing. If $T=1$, then $f'(1)=\frac{dT(1)}{dt}=-2r-\mu$ thus $f'(1)$ is negative. Therefore when $T=1$, then $f=T$ is decreasing.

Therefore, the model is well defined or in other words the model is feasible to use. Equation (3.4) and equation (3.5) are the final equations to be analyzed..

B. Analysis Results

1. Fixed Point of the Model

Fixed point analysis in systems of differential equations is frequently used to find solutions invariable of time. Based on the definition (7), the fixed point of the model can be determined when $\frac{dK}{dt}=0, \frac{dT}{dt}=0$. Thus, it is obtained:

$$jKT+p-pK-pT+z-zK-zT-rK-\mu K=0 \qquad (3.6)$$

$$4p-4pK-4pT+z-zK-zT-2rT-jKT-\mu T=0 \qquad (3.7)$$

From Equations (3.6) and (3.7) the fixed point can be obtained as follows:

From Equations (3.6) it is obtained:

$$T = \frac{K\mu + Kp + Kr + Kz - p - z}{Kj - p - z} \tag{3.8}$$

From Equations (3.7) it is obtained:

$$K = -\frac{T\mu + 4Tp + 2Tr + Tz - 4p - z}{Tj + 4p + z} \tag{3.9}$$

Subtituting Equation (3.9) into (3.8),

$$T = \frac{\left(\left(-\frac{T\mu + 4Tp + 2Tr + Tz - 4p - z}{Tj + 4p + z}\right)(\mu + p + r + z)\right) - p - z}{\left(-\frac{T\mu + 4Tp + 2Tr + Tz - 4p - z}{Tj + 4p + z}\right)j - p - z}$$

Thus,

$$\begin{aligned} T = -\frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)} \Big(&-\mu^2 - 5\mu p - 3\mu r - 2\mu z - 5jp - 2jz - 6pr \\ &- 2r^2 - 3rz \\ &+ (\mu^4 + 10\mu^3 p + 6\mu^3 r + 4\mu^3 z - 6\mu^2 jp + 25\mu^2 p^2 + 42\mu^2 pr \\ &+ 20\mu^2 pz + 13\mu^2 r^2 + 18\mu^2 rz + 4\mu^2 z^2 - 30\mu jp^2 - 18\mu jpr \\ &- 12\mu jpz + 60\mu p^2 r + 56\mu pr^2 + 54\mu prz + 12\mu r^3 + 26\mu r^2 z \\ &+ 12\mu rz^2 + 25j^2 p^2 + 20j^2 pz + 4j^2 z^2 - 20jpr^2 - 12jpr^2 \\ &+ 2jprz + 4jrz^2 + 36p^2 r^2 + 24pr^3 + 36pr^2 z + 4r^2 + 12r^3 z \\ &+ 9r^2 z^2)^{1/2}\Big) \end{aligned}$$

Or

$$T = \frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)}\Bigg[(3\mu + 6p + 3z)r + \mu^2 + 2r^2 + (5p + 2z)\mu$$
$$+ (5p + 2z)j$$
$$- \Bigg((25\mu^2 + (-30j + 60r)\mu + 25j^2 - 20jr + 36r^2)p^2$$
$$+ \Bigg((20\mu^2 + (-12j + 54r)\mu + 20j^2 + 2jr + 36r^2)z$$
$$+ 10(\mu + r)(\mu + 2r)\left(\mu - \frac{3}{5}j + \frac{6}{5}r\right)\Bigg)p$$
$$+ (4\mu^2 + 12\mu r + 4j^2 + 4jr + 9r^2)z^2$$
$$+ 4(\mu + r)(\mu + 2r)\left(\frac{3}{2}r + \mu\right)z + (\mu + 2r)^2(\mu + r)^2\Bigg)^{1/2}\Bigg]$$

Subtituting Equation (3.8) into (3.9):

$$K = -\frac{\left(\frac{K\mu + Kp + Kr + Kz - p - z}{Kj - p - z}\right)(\mu + 4p + 2r + z) - 4p - z}{\left(\frac{K\mu + Kp + Kr + Kz - p - z}{Kj - p - z}\right)j + 4p + z}$$

Thus,

$$K = -\frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)}\Big(-\mu^2 - 5\mu p - 3\mu r - 2\mu z + 5jp + 2jz + 6pr - 2r^2$$
$$- 3rz$$
$$+ (\mu^4 + 10\mu^3 p + 6\mu^3 r + 4\mu^3 z - 6\mu^2 jp + 25\mu^2 p^2 + 42\mu^2 pr$$
$$+ 20\mu^2 pz + 13\mu^2 r^2 + 18\mu^2 rz + 4\mu^2 z^2 - 30\mu jp^2 - 18\mu jpr - 12\mu jpz$$
$$+ 60\mu p^2 r + 56\mu pr^2 + 54\mu prz + 12\mu r^3 + 26\mu r^2 z + 12\mu rz^2$$
$$+ 25j^2 p^2 + 20j^2 pz + 4j^2 z^2 - 20jpr^2 - 12jpr^2 + 2jprz + 4jrz^2$$
$$+ 36p^2 r^2 + 24pr^3 + 36pr^2 z + 4r^2 + 12r^3 z + 9r^2 z^2)^{1/2}\Big)$$

Or

$$K = \frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)}\left[(-3\mu - 6p - 3z)r - \mu^2 - 2r^2 + (-5p - 2z)\mu\right.$$

$$+ (5p + 2z)j$$

$$+ \Bigg((25\mu^2 + (-30j + 60r)\mu + 25j^2 - 20jr + 36r^2)p^2$$

$$+ \Bigg((20\mu^2 + (-12j + 54r)\mu + 20j^2 + 2jr + 36r^2)z$$

$$+ 10(\mu + r)(\mu + 2r)\left(\mu - \frac{3}{5}j + \frac{6}{5}r\right)\Bigg)p$$

$$+ (4\mu^2 + 12\mu r + 4j^2 + 4jr + 9r^2)z^2 + 4(\mu + r)(\mu + 2r)\left(\frac{3}{2}r + \mu\right)z$$

$$\left. + (\mu + 2r)^2(\mu + r)^2\Bigg)^{1/2}\right]$$

Then a fixed point (K^*, T^*) is obtained, where:

$$K^* = \frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)}\left[(-3\mu - 6p - 3z)r - \mu^2 - 2r^2 + (-5p - 2z)\mu\right.$$

$$+ (5p + 2z)j$$

$$+ \Bigg((25\mu^2 + (-30j + 60r)\mu + 25j^2 - 20jr + 36r^2)p^2$$

$$+ \Bigg((20\mu^2 + (-12j + 54r)\mu + 20j^2 + 2jr + 36r^2)z$$

$$+ 10(\mu + r)(\mu + 2r)\left(\mu - \frac{3}{5}j + \frac{6}{5}r\right)\Bigg)p$$

$$+ (4\mu^2 + 12\mu r + 4j^2 + 4jr + 9r^2)z^2 + 4(\mu + r)(\mu + 2r)\left(\frac{3}{2}r + \mu\right)z$$

$$\left. + (\mu + 2r)^2(\mu + r)^2\Bigg)^{1/2}\right]$$

$$T^* = \frac{1}{2}\frac{1}{j(\mu+5p+2r+2z)}\Bigg[(3\mu+6p+3z)r+\mu^2+2r^2+(5p+2z)\mu+(5p+2z)j$$

$$-\Bigg((25\mu^2+(-30j+60r)\mu+25j^2-20jr+36r^2)p^2$$

$$+\Bigg((20\mu^2+(-12j+54r)\mu+20j^2+2jr+36r^2)z$$

$$+10(\mu+r)(\mu+2r)\left(\mu-\frac{3}{5}j+\frac{6}{5}r\right)\Bigg)p$$

$$+(4\mu^2+12\mu r+4j^2+4jr+9r^2)z^2+4(\mu+r)(\mu+2r)\left(\frac{3}{2}r+\mu\right)z$$

$$+(\mu+2r)^2(\mu+r)^2\Bigg)^{1/2}\Bigg]$$

2. Fixed point stability test in various cases

a. Jacobian matrix

The Jacobi matrix system of equations for the mathematical model of zakat in poverty reduction is formed using Definition (6) as follows:

$$J(f(x)) = \begin{pmatrix} \frac{df_1}{dK} & \frac{df_1}{dT} \\ \frac{df_2}{dK} & \frac{df_2}{dT} \end{pmatrix}$$

The entries for Jacobian matrix above is

$$f_1(K,T) = jKT + p - pK - pT + z - zK - zT - rK - \mu K$$

$$f_2(K,T) = 4p - 4pK - 4pT + z - zK - zT - 2rT - jKT - \mu T$$

By setting:

$$\frac{dK}{dt} = f_1, \qquad \frac{dT}{dt} = f_2$$

It is obtained a Jacobian matrix for the system of differential equations above containing the following entries:

1) Obtained $\frac{df_1(K,T)}{dK} = jT - p - z - r - \mu$
2) Obtained $\frac{df_1(K,T)}{dT} = jK - p - z$
3) Obtained $\frac{df_2(K,T)}{dK} = -4p - z - jT$
4) Obtained $\frac{df_2(K,T)}{dT} = -4p - z - 2r - jK - \mu$

as written as follows:

$$J\big(f(x)\big) = \begin{pmatrix} jT - p - z - r - \mu & jK - p - z \\ -4p - z - jT & -4p - z - 2r - jK - \mu \end{pmatrix} \quad (3.10)$$

b. Fixed point stability test

In order to determine the stability of a fixed point the system, the eigen value of the Jacobi matrix is calculated as follows

$$J\big(f(x)\big) = \begin{pmatrix} jT - p - z - r - \mu & jK - p - z \\ -4p - z - jT & -4p - z - 2r - jK - \mu \end{pmatrix}$$

A number of parameter values are to be used to test the stability of fixed point in the following 12 cases:

Case 1: $j = 0.01;\ \ p = 0.04;\ \ z = 0.01;\ \ r = 0.008;\ \ \mu = 0.014$

The Jacobian entries for this case are as follows:

$$\frac{df_1(K,T)}{dK} = (0.01)(0.55) - 0.04 - 0.01 - 0.008 - 0.014$$

$$\frac{df_1(K,T)}{dT} = (0.01)(0.33) - 0.04 - 0.01$$

$$\frac{df_2(K,T)}{dK} = -4(0.04) - 0.01 - (0.01)(0.55)$$

$$\frac{df_2(K,T)}{dT} = -4(0.04) - 0.01 - 2(0.008) - (0.01)(0.33) - 0.014$$

Hence, the following Jacobian matrix is obtained

$$J(E) = \begin{pmatrix} -0.0665 & -0.0467 \\ -0.1755 & -0.2033 \end{pmatrix}$$

solving $det(\lambda I - J(E)) = 0$, the eigen values for $J(E)$ are obtained,

$$|\lambda I - J(E)| = \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} -0.0665 & -0.0467 \\ -0.1755 & -0.2033 \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} \lambda + 0.0665 & 0.0467 \\ 0.1755 & \lambda + 0.2033 \end{pmatrix} \right| = 0$$

$$\big((\lambda + 0.0665)(\lambda + 0.2033)\big) - \big((0.0467)(0.1755)\big) = 0$$

$$\lambda^2 + 0.2698\lambda + 0.00532360 = 0$$

$$(\lambda_1 + 0.0214)(\lambda_2 + 0.2483) = 0$$

$$\lambda_1 = -0.0214, \ \lambda_2 = -0.2483$$

By using the same method, the fixed point and the eigen values for the other parameters will be obtained,

$$\mu = 0.014$$

Kasus	z	j	r	p	Titik tetap (K^*, T^*)	Q^*	Nilai eigen
Kasus 1	0.01	0.01	0.008	0.04	$(0.33, 0.55)$	0.12	$(-0.0214, -0.2483)$
Kasus 2				0.06	$(0.33, 0.58)$	0.09	$(-0.0230, -0.3464)$
Kasus 3				0.08	$(0.34, 0.59)$	0.07	$(-0.0211, -0.4483)$
Kasus 4	0.01	0.01	0.009	0.04	$(0.29, 0.57)$	0.14	$(-0.0226, -0.1341)$
Kasus 5			0.011		$(0.30, 0.56)$	0.14	$(-0.0249, -0.2501)$
Kasus 6			0.013		$(0.30, 0.58)$	0.12	$(-0.0267, -0.2536)$
Kasus 7	0.01	0.02	0.008	0.04	$(0.43, 0.43)$	0.14	$(-0.0217, -0.2454)$
Kasus 8		0.04			$(0.57, 0.30)$	0.13	$(-0.0152, -0.2496)$
Kasus 9		0.06			$(0.67, 0.21)$	0.12	$(-0.0470, -0.2495)$
Kasus 10	0.02	0.01	0.008	0.04	$(0.36, 0.53)$	0.11	$(-0.0221, -0.2681)$
Kasus 11	0.04				$(0.40\ 0.50)$	0.10	$(-0.0238, -0.3081)$
Kasus 12	0.06				$(0.43, 0.49)$	0.08	$(-0.0239, -0.3476)$

Because in all cases the eigenvalues show a negative sign, it can be concluded that all fixed points in all cases are stable.

3. Fixed point stability simulation

The simulation of the stability of the fixed point mathematical model of zakat for poverty reduction is performed by setting up a trajectory from different initial conditions. From the resulting trajectory, it can be seen that if the curve leads to a fixed point at a certain time, it can be concluded that the fixed point exists and is true. In other words, simulating the stability of a fixed point is also used as a tool to test the existence of a fixed point.

Simulations of the stability of the fixed point of zakat are carried out to see the trajectories around this fixed point with several different parameter values used as follows:

a) **Case 9:** $j = 0.01$; $p = 0.04$; $z = 0.01$; $r = 0.008$; $\mu = 0.014$

The following three initial values are used:

$K(0) = 0.48; T(0) = 0.40$

$K(0) = 0.61; T(0) = 0.27$

$K(0) = 0.73; T(0) = 0.15$

Based on the initial values above, the chart of each class versus time t is obtained as follows:

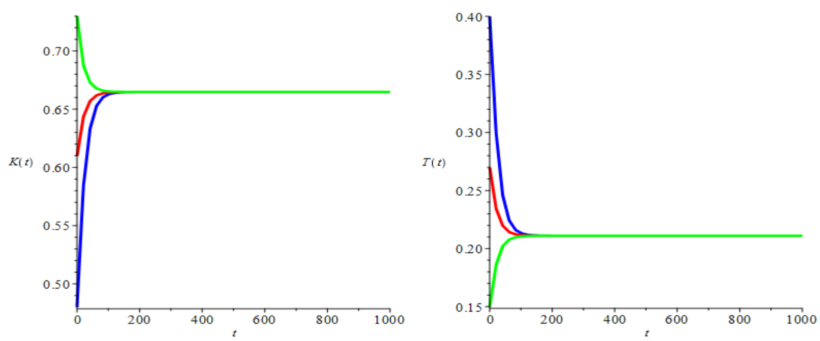

Chart for $K(t)$ **Chart for $T(t)$**

Figure 3.2 Trajectory around the fixed point for Case 9

The fixed point $E = (0.67, 0.21)$ is obtained if parameters are subtituted. From Figure 3.2 it can be seen that the direction of motion of the curve is both approaching a fixed point. Therefore, the fixed point obtained is stable.

b) **Case 12**: $j = 0.01$; $p = 0.004$; $z = 0.06$; $r = 0.008$; $\mu = 0.014$

The following intial values are used:

$K(0) = 0.48; T(0) = 0.44$

$K(0) = 0.38; T(0) = 0.54$

$K(0) = 0.73; T(0) = 0.19$

The chart of each class versus time t is obtained as follows:

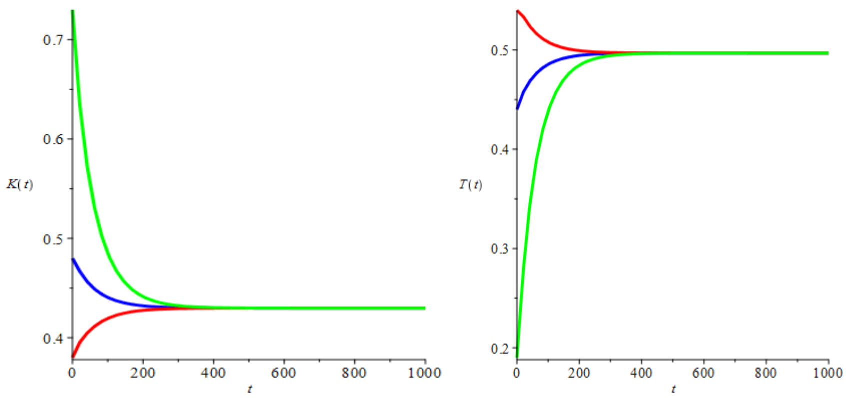

Chart for $K(t)$ Chart for $T(t)$

Figure 3.3 Trajectory around the fixed point for Case 12

If all parameters are substituted then the fixed point $E = (0.43, 0.49)$ is obtained. From Figure 3.3 it can be seen that the direction of motion of the curve is both approaching a fixed point. So that the fixed point obtained is stable. Using the same method for Case 9 and Case 12 for the other remaining cases, it is quite simple to find out that the remaining fixed points are also stable.

C. Model Interpretation

Based on the results discussed above, a fixed point $E = (K^*, T^*)$ is obtained. Calculating for $Q^* = 1 - K^* - T^*$, it is obtained all the following variables K^*, T^*, Q^*, namely:

$$K^* = \frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)}\left[(-3\mu - 6p - 3z)r - \mu^2 - 2r^2 + (-5p - 2z)\mu \right.$$

$$+ (5p + 2z)j$$

$$+ \left((25\mu^2 + (-30j + 60r)\mu + 25j^2 - 20jr + 36r^2)p^2 \right.$$

$$+ \left((20\mu^2 + (-12j + 54r)\mu + 20j^2 + 2jr + 36r^2)z \right.$$

$$\left. + 10(\mu + r)(\mu + 2r)\left(\mu - \frac{3}{5}j + \frac{6}{5}r\right)\right)p$$

$$+ (4\mu^2 + 12\mu r + 4j^2 + 4jr + 9r^2)z^2$$

$$\left. \left. + 4(\mu + r)(\mu + 2r)\left(\frac{3}{2}r + \mu\right)z + (\mu + 2r)^2(\mu + r)^2\right)^{1/2}\right]$$

$$T^* = \frac{1}{2}\frac{1}{j(\mu+5p+2r+2z)}\Bigg[(3\mu+6p+3z)r+\mu^2+2r^2+(5p+2z)\mu+(5p+2z)j$$

$$-\Bigg((25\mu^2+(-30j+60r)\mu+25j^2-20jr+36r^2)p^2$$

$$+\Bigg((20\mu^2+(-12j+54r)\mu+20j^2+2jr+36r^2)z$$

$$+10(\mu+r)(\mu+2r)\left(\mu-\frac{3}{5}j+\frac{6}{5}r\right)\Bigg)p$$

$$+(4\mu^2+12\mu r+4j^2+4jr+9r^2)z^2+4(\mu+r)(\mu+2r)\left(\frac{3}{2}r+\mu\right)z$$

$$+(\mu+2r)^2(\mu+r)^2\Bigg)^{1/2}\Bigg]$$

$$Q^* = \frac{1}{2}\frac{1}{j(\mu+5p+2r+2z)(\mu+5p+r+2z)}(2r^3+(3\mu+4j+6p+3z)r^2$$

$$+\Bigg(\mu^2+(6j+5p+2z)\mu+(15p+6z)j$$

$$-\Bigg(\Bigg(25\mu^2+(-30j+60r)\mu+25j^2-20jr+36r^2)p^2$$

$$+\Bigg((20\mu^2+(-12j+54r)\mu+20j^2+2jr+36r^2)z$$

$$+10(\mu+r)\left(\mu-\frac{3}{5}j+\frac{6}{5}r\right)(\mu+2r)\Bigg)p$$

$$+(4\mu^2+12\mu r+4j^2+4jr+9r^2)z^2+4(\mu+r)\left(\frac{3}{2}r+\mu\right)(\mu+2r)z$$

$$+(\mu+2r)^2(\mu+r)^2\Bigg)^{1/2}\Bigg)r+2\mu j(\mu+5p+2z)\Bigg)$$

The factors that affects zakat in reducing poverty are the birth rate (μ), the rate of interaction (j), the failure rate in running a business (r), the rate of success in running a business which is indicated by economic growth (p), the zakat effect on economic growth (z).

The strongest factor found is the zakat effect on economic growth. Since the zakat is managed by a zakat management agency which is expected to have a number of superior programs, the zakat distribution to recipients is considered to be more effective and accurate. This may promote micro growth in the economy of the poor's business and household consumption, which simultaneously leads to macro economic growth. The other mechanism ozf zakat to improve the economy of the poor is by providing aids on education (scholarship, regular assistance on tuition) in order for the unlucky children to have better future. This is an indirect way the zakat for reducing the poverty in long term.

From the analysis already carried out, the rate of zakat effect on reducing g poverty takes a realtively longer time to achieve stability compared to the other factor that also reduces poverty i.e the rate of interaction.

The other effective factor is the economic growth affecting the business of zakat recipients without the assistance of zakat. It is pure economic purchasing power. In addition to that, the low rate of business failure also affects changes in the number of poor people. The low failure rate in running a business prevents an increase in the number of zakat recipients as well as other poor people.

By comparing what have been found within each observed group by economy, it is revealed that a high rate of interaction between true and discouraged zakat debtors leads to a reduction in the poor in a shorter time compared to other factors which can also reduce poverty but take a longer time.

Thus, the main suggestion from the results is that zakat is ideally effective for reducing poverty over time considering factors and assumptions involved in this study. If such factors are controllable, then the effect of zakat on reducing the poverty may become much more optimal.

CHAPTER IV
SUMMARY

A. Conclusion

Based on the results have been discussed in the previous chapter, it is concluded that the mathematical model of zakat for poverty reduction is a nonlinear differential equation system which can be written as follows:

$$\frac{dK}{dt} = jKT + pQ + zQ - rK - \mu K$$

$$\frac{dT}{dt} = 4pQ + zQ - 2rT - jKT - \mu T$$

$$\frac{dQ}{dt} = 2rT + rK - 5pQ - 2zQ - \mu Q + \mu$$

where:

$\bar{K}$: the number of true zakat debtors (obliged and pay)

$\bar{T}$: the number of discouraged zakat debtors (obliged but do not pay)

$\bar{Q}$: the number of zakat recipients

Thus, the total population is

$$N = \bar{K} + \bar{T} + \bar{Q}$$

The parameters used are as follows:

j : interaction rate between true and discouraged zakat debtors

p : business success rate indicated by economic growth

r : business failure rate

z : zakat effect on economy

μ : natural birth and mortality rate

1. The fixed point of the model obtained:

$$E = (K^*, T^*)$$

where:

$$K^* = \frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)}\Bigg[(-3\mu - 6p - 3z)r - \mu^2 - 2r^2 + (-5p - 2z)\mu + (5p + 2z)j$$

$$+ \Bigg((25\mu^2 + (-30j + 60r)\mu + 25j^2 - 20jr + 36r^2)p^2$$

$$+ \Bigg((20\mu^2 + (-12j + 54r)\mu + 20j^2 + 2jr + 36r^2)z$$

$$+ 10(\mu + r)(\mu + 2r)\left(\mu - \frac{3}{5}j + \frac{6}{5}r\right)\Bigg)p + (4\mu^2 + 12\mu r + 4j^2 + 4jr + 9r^2)z^2$$

$$+ 4(\mu + r)(\mu + 2r)\left(\frac{3}{2}r + \mu\right)z + (\mu + 2r)^2(\mu + r)^2\Bigg)^{1/2}\Bigg]$$

$$T^* = \frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)}\Bigg[(3\mu + 6p + 3z)r + \mu^2 + 2r^2 + (5p + 2z)\mu + (5p + 2z)j$$

$$- \Bigg((25\mu^2 + (-30j + 60r)\mu + 25j^2 - 20jr + 36r^2)p^2$$

$$+ \Bigg((20\mu^2 + (-12j + 54r)\mu + 20j^2 + 2jr + 36r^2)z$$

$$+ 10(\mu + r)(\mu + 2r)\left(\mu - \frac{3}{5}j + \frac{6}{5}r\right)\Bigg)p + (4\mu^2 + 12\mu r + 4j^2 + 4jr + 9r^2)z^2$$

$$+ 4(\mu + r)(\mu + 2r)\left(\frac{3}{2}r + \mu\right)z + (\mu + 2r)^2(\mu + r)^2\Bigg)^{1/2}\Bigg]$$

$E = (K^*, T^*)$ is a stable fixed point. Having $Q^* = 1 - K^* - T^*$ it is obtained:

$$Q^* = \frac{1}{2}\frac{1}{j(\mu + 5p + 2r + 2z)(\mu + 5p + r + 2z)}(2r^3 + (3\mu + 4j + 6p + 3z)r^2$$

$$+ \Bigg(\mu^2 + (6j + 5p + 2z)\mu + (15p + 6z)j$$

$$- \Bigg(\Bigg(25\mu^2 + (-30j + 60r)\mu + 25j^2 - 20jr + 36r^2)p^2$$

$$+ \Bigg((20\mu^2 + (-12j + 54r)\mu + 20j^2 + 2jr + 36r^2)z$$

$$+ 10(\mu + r)\left(\mu - \frac{3}{5}j + \frac{6}{5}r\right)(\mu + 2r)\Bigg)p$$

$$+ (4\mu^2 + 12\mu r + 4j^2 + 4jr + 9r^2)z^2 + 4(\mu + r)\left(\frac{3}{2}r + \mu\right)(\mu + 2r)z$$

$$+ (\mu + 2r)^2(\mu + r)^2\Bigg)^{1/2}\Bigg)r + 2\mu j(\mu + 5p + 2z)\Bigg)$$

2. The interpretation of the mathematical model of zakat in poverty reduction is that it can reduce poverty when zakat takes role as a business capital that may promote economic growth, in addition to the rate of business success (as an effect of pure economic growth). This mechanism may reduce the number of zakat recipients which potentitally come from the group of poor people. The rate of interaction can also reduce poverty in relatively shorter time since this factor is found to be faster moving towards to stability than other factors.

B. Recommendations

This study has not been considered the the existence of an economic group of those not obliged to zakat and does not also receive zakat (mediocre economy) in the model assumptions. Therefore, it remains a gap the can be further studied in the future since it may also affect the process of reducing the number of poor people.

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APPENDIX

Appendix 1. MAPLE™ syntax in the simulation to find the fixed point for Case 9

> *restart*;

>

sistem := [*diff*(*K*(*t*), *t*) = *j*·*K*·*T*(*t*) + *p* − *p*·*K*(*t*) − *p*·*T*(*t*) + *z* − *z*·*K*(*t*) − *z*·*T*(*t*) − *r*·*K* − *m* ·*K*(*t*), *diff*(*T*(*t*), *t*) = 4·*p* − 4·*p*·*K*(*t*) − 4·*p*·*T*(*t*) + *z* − *z*·*K*(*t*) − *z*·*T*(*t*) − 2·*r*·*T*(*t*) − *j* ·*K*·*T*(*t*) − *m*·*K*(*t*)];

> *dsolve*[':-*interactive*']

> *parameter1* := {*p* = 0.04, *r* = 0.008, *z* = 0.01, *j* = 0.06, *m* = 0.014};

> *A* := *subs*(*parameter1*, *sistem*);

> *dsolve*[':-*interactive*'

> *with*(*DEtools*) :

>

DEplot(*A*, [*K*(*t*), *T*(*t*)], *t* = 0 ..1000, [[*K*(0) = 0.48, *T*(0) = 0.40], [*K*(0) = 0.61, *T*(0) = 0.27], [*K*(0) = 0.73, *T*(0) = 0.15]], *scene* = [*t*, *K*(*t*)], *linecolour* = [*blue*, *red*, *green*]);

> *with*(*DEtools*) :

>

DEplot(*A*, [*K*(*t*), *T*(*t*)], *t* = 0 ..1000, [[*K*(0) = 0.48, *T*(0) = 0.40], [*K*(0) = 0.61, *T*(0) = 0.27], [*K*(0) = 0.73, *T*(0) = 0.15]], *scene* = [*t*, *T*(*t*)], *linecolour* = [*blue*, *red*, *green*]);

Appendix 2. MAPLE™ syntax in the simulation to find the fixed point for Case 12

> *restart*;

>

$sistem := [diff(K(t), t) = j \cdot K \cdot T(t) + p - p \cdot K(t) - p \cdot T(t) + z - z \cdot K(t) - z \cdot T(t) - r \cdot K - m \cdot K(t), diff(T(t), t) = 4 \cdot p - 4 \cdot p \cdot K(t) - 4 \cdot p \cdot T(t) + z - z \cdot K(t) - z \cdot T(t) - 2 \cdot r \cdot T(t) - j \cdot K \cdot T(t) - m \cdot K(t)];$

> *dsolve*[':-*interactive*']

> $parameter1 := \{p = 0.04, r = 0.008, z = 0.06, j = 0.01, m = 0.014\};$

> $A := subs(parameter1, sistem);$

> *dsolve*[':-*interactive*']

> *with*(*DEtools*) :

>

$DEplot(A, [K(t), T(t)], t = 0 ..1000, [[K(0) = 0.48, T(0) = 0.44], [K(0) = 0.38, T(0) = 0.54], [K(0) = 0.73, T(0) = 0.19]], scene = [t, K(t)], linecolour = [blue, red, green]);$

> *with*(*DEtools*) :

>

$DEplot(A, [K(t), T(t)], t = 0 ..1000, [[K(0) = 0.48, T(0) = 0.44], [K(0) = 0.38, T(0) = 0.54], [K(0) = 0.73, T(0) = 0.19]], scene = [t, T(t)], linecolour = [blue, red, green]);$

Appendix 3. Proof for eigen values

Consider the multiplication of the following matrix A and vector X in the Definition

5

$$AX = \lambda X$$

Multiplying both hands with identity matrix:

$$IAX = I\lambda X$$

$$AX = \lambda IX$$

$$(\lambda I - A)X = 0 \qquad (1)$$

Since X is a non zero vector then Eq. (1) is solvable if and only if $det(\lambda I - A) = 0$